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Composite Dark Matter from Non-Abelian Gauge Theories

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Introduction

The Standard Model of Fundamental Interactions has been extremely successful in explaining particle physics observations up to energies of a few TeV. Yet, astrophysical and cosmological data suggest that more than 80% of the matter energy density in the Universe cannot be accounted for in terms of ordinary matter. In this work we consider extensions of the Standard Model based on the same principles that justify its effectiveness and featuring accidentally stable composite dark matter candidates.

The success of the Standard Model in describing all the observed microscopic phenomena can be understood if it is considered as an effective field theory with an high ultraviolet cut-off. Higher dimensional non-renormalizable operators are suppressed by powers of the cut-off scale and become irrelevant in the infrared. The renormalizable lagrangian features some accidental global symmetries, such as baryon and lepton number conservation, and custodial symmetry; these give a natural explanation to many experimental observations. A fundamental ingredient of the Standard Model is its gauge theory structure, which provides a rich infrared dynamics while giving, nonetheless, a renormalizable field theory.

We consider such properties as paradigmatic, offering a theoretical rational for the success of the Standard Model, and try to use them as guidance principles to build possible extensions featuring a dark matter candidate.

We focus on extensions of the Standard Model based on a new non-Abelian gauge interaction (with gauge group $\mathcal{G}_{\mathcal{DC}}$) and new fermionic fields (*dark quarks*) charged under both $\mathcal{G}_{\mathcal{DC}}$ and $\mathcal{G}_{\mathcal{SM}}$. We refer to the field content added to the Standard Model as the *dark sector*. Differently from technicolor and composite Higgs models, we require that the dark sector dynamics does not break the Standard Model gauge group.

Previous works have studied QCD-like models which have accidentally stable baryon-like states that can account for the dark matter [1, 2]. The purpose of this thesis is to generalise this construction and understand if there are different scenarios which, despite being based on the same framework, feature a different dynamics and phenomenology.

As a first step, we critically analyse the implicit assumptions of these models and suggest possible generalisations, identifying some interesting alternatives which have received little attention.

An interesting class of models is that of chiral models, based on complex representations of the gauge group, in which the fermionic fields transform in a complex (reducible) representation of the gauge group. After discussing some general properties and what are the necessary conditions to have a consistent model and preserve $\mathcal{G}_{\mathcal{SM}}$, we focus on the case $\mathcal{G}_{\mathcal{DC}} = \mathrm{SU}(N)_{\mathrm{DC}}$ and argue on general ground, using 't Hooft anomaly matching, that in the absence of fundamental scalar fields these models have always light states. This considerations are compared with the case of the Standard Model, which is itself a chiral theory, giving a different viewpoint on some well-known facts.

We then consider models based on real representations and focus on the scenario in which the dark colour gauge group dynamics has an infrared fixed point, up to deformations induced by fermions mass terms. We discuss general model building issues and how the would-be conformal symmetry is broken by the mass terms, inducing a confining dynamics. We identify, in particular, a model with a perturbative infrared fixed point and consider two possible assignments of Standard Model quantum numbers as benchmark scenarios. The dark sector field content corresponds to five Weyl fermions, each one transforming as the adjoint of the gauge group $\mathcal{G}_{\mathcal{DC}} = \mathrm{SU}(3)_{\mathrm{DC}}$, together with the gauge bosons (*dark gluons*). We discuss the dynamics of these models, with an emphasis on asymptotic states below the confinement scale (gluonium bound states, *i.e. glueballs*, and gluon-quark bound states, *i.e. gluequarks*) and their accidental stability.

As a last step, we study the phenomenology of the two models, with particular attention to the consistency with cosmological observations. Assuming that all the dark quarks have the same mass, the model can be characterised by two scales, namely the confinement scale Λ_{DC} and the dark quark mass $M_{\mathcal{Q}}$. A natural hierarchy of scales arises from the structure of the model and we study the different phenomenological regimes as a function of the two scales.

After estimating the glueball lifetime with an effective field theory approach, we identify two relevant regimes for the models: one in which the glueballs are stable on cosmological scales and one in which they decay quickly.

The first scenario has a non-standard cosmology with so called *cannibalism* in the glueball sector (*i.e.* number changing interactions involving only glueballs, such as $3 \to 2$ processes). This translates into an unusual scale dependence of the dark sector temperature. We evaluate the glueballs relic density and the cosmological bounds, concluding that this parameter space region is excluded by observations.

In the second scenario the only stable relics are gluequarks. To evaluate their relic density, we compute the cross section for the annihilation of dark quarks in dark gluons, including the Sommerfeld enhancement correction. In order to reproduce the dark matter relic abundance, dark quarks with mass in the range $M_{\mathcal{Q}} = (1 \div 10)\,\mathrm{TeV}$ are needed.

The thesis is organised as follows. Chapter 1 and 2 are devoted to presenting the motivations and the aims of this work. In the first one we discuss the reasons for going beyond the Standard Model, briefly reviewing the status of the theory, with a particular emphasis on the evidences and properties of dark matter. In chapter 2 we discuss what are the motivations for considering composite dark matter models based on non-Abelian gauge theories and than provide an overview of the framework we shall be considering, identifying the purposes of the thesis in relation to the literature. The original content of the thesis is presented in the chapters 3, 4 and 5, together with some background material. The original results include an analysis of some dynamical properties of chiral models (chapter 3), the introduction and study of two possible models for composite dark matter with an infrared fixed point (chapter 4) and the study of the phenomenology of the two models previously introduced, taking into account the cosmological constraints (chapter 5). In the conclusions we summarise our results and give an outlook of the possible extensions of this work.

Chapter 1

Motivations

Particle physics and its connections to cosmology offer some of the most pressing unanswered questions in modern science.

In what follows we shall present a short summary of the current status of the theory, explaining why we believe that the Standard Model is not a complete theory. Afterwards, we focus on the problem of dark matter and outline the main observational evidences and what we know about its properties.

1.1 Successes and limitations of the Standard Model

The Standard Model of Fundamental Interactions offers an accurate description of the elementary constituents of matter[1]. It is a quantum field theory based on the gauge group $\mathcal{G}_{\mathcal{SM}} = \mathrm{SU}(3)_\mathrm{c} \times \mathrm{SU}(2)_{EW} \times \mathrm{U}(1)_Y$, broken to $\mathrm{SU}(3)_\mathrm{c} \times \mathrm{U}(1)_\mathrm{em}$ through the Higgs mechanism. Its fermionic field content can be described through the following irreducible representations of $\mathcal{G}_{\mathcal{SM}}$:

$$Q_L = \left(3, 2, \frac{1}{6}\right) \oplus u_R = \left(3, 1, \frac{2}{3}\right) \oplus d_R = \left(3, 1, -\frac{1}{3}\right) \oplus L_L = \left(1, 2, -\frac{1}{2}\right) \oplus e_R = \left(1, 1, -1\right)$$

where $\psi_{L(R)}$ denotes a left-handed (right-handed) spinor, and hypercharge is chosen to satisfy the normalization $Q_{em} = T_3 + Y$. In addition to the fermions, the Standard Model includes a complex scalar field (Higgs field), transforming as

$$H = \left(1, 2, \frac{1}{2}\right)$$

H has a non-zero vacuum expectation value, realising the Higgs mechanism which leads to the breaking of the Standard Model gauge group.

1.1.1 Status

The discovery of the Higgs boson in 2012 by the ATLAS and CMS collaborations operating at the LHC represents only the last of a long series of experimental successes.

[1] Elementary up to a distance scale of order $10^{-18}\,\mathrm{m}$

Quantum Chromodynamics explains satisfactorily the structure of mesons and baryons, their classification and their interactions, as well as all the observations in hadronic physics. Although the theory is non-perturbative in the low energy regime, the use of effective field theories (such as chiral perturbation theory) and the development of non-perturbative methods (such as lattice simulations) have provided an accurate description of the phenomenology. Asymptotic freedom, confinement and chiral symmetry breaking are properties which are well accounted by this description and verified experimentally. Moreover, the theoretical study of the non-perturbative structure of gauge theories through semiclassical methods (such as instantons) and the development of alternative perturbative expansions (such as the large N_c expansion), have offered a more complete and profound point of view on many phenomenological aspects.

The Standard Model is completely predictive also for what concerns flavour physics. All the observed flavour violation effects can be explained in terms of the CKM matrix parameters: three mixing angles and one complex phase. The existence of the charm quark, conjectured the explain the suppression of *flavour changing neutral currents*, and that of a third generation of quarks, necessary to account for the CP symmetry violation in the weak interactions, have been two among the most well known theoretical prediction of the Standard Model that have been experimentally verified.

Lastly, the electroweak sector describes in a unified setting the electromagnetic and weak interactions. The observation of neutral currents and, later, of the massive vector bosons W and Z have confirmed experimentally the Weinberg-Salam model of electroweak interactions based on the pattern of symmetry breaking $\mathrm{SU}(2)_\mathrm{EW} \times \mathrm{U}(1)_\mathrm{Y} \to \mathrm{U}(1)_\mathrm{em}$. Nevertheless, it has been only with the discovery of the Higgs boson in 2012 that the dynamical mechanism at the origin of the electroweak symmetry breaking has been probed directly. All the experimental results, such as interaction vertices with the gauge bosons and Yukawa coupling with fermions, agree with the simple hypothesis of an Higgs field (complex scalar doublet). Altogether, electroweak precision tests put strong constraints on the eventuality of a new dynamics at the TeV scale.

1.1.2 Problems

Despite the many successes of the Standard Model, there are several aspects that appear to be not satisfactory from a theoretical point of view and some experimental observations are not explained by the model.

From a theoretical perspective:

- There are strong indication in favour of an algebraical unification of the gauge group. For instance, considering the group $\mathrm{SU}(5)$ and properly identifying some of its subgroups with the factor of the Standard Model gauge group $\mathcal{G}_{\mathcal{SM}}$, it is possible to organise all the fermionic fields in just two irreducible representations: $\bar{5} \oplus 10$, with a multiplicity of 3 (number of generations). Studying the evolution of the gauge coupling constants at high energies under the renormalization group flow, one can show that they become comparable at an energy scale of order 10^{15} GeV. However the unification is not completely realised and the extension of the Higgs mechanism presents some problems.

- Gravity can be included in the Standard Model, which however becomes non-renormalizable. This limits the regime of validity of the theory, which has an ultraviolet cut-off at the scale of the Planck mass $M_{\rm Pl} \sim 10^{19}$ GeV; beyond this scale an UV completion is needed.

- The hypercharge gauge group $\mathrm{U}(1)_Y$ is not asymptotically free and perturbativity is lost at energy scales near its Landau pole. In general one would expect a UV completion, unless the theory is proven to be asymptotically safe.

- The mass term in the Higgs potential has positive energy dimension and is not protected by any symmetry. In a generic ultraviolet completion of the Standard Model in which the mass of the Higgs boson in the effective IR theory is calculable, we expect terms proportional to the UV cut-off scale. In order to have a light Higgs in the infrared theory it is necessary to have very precise cancellations in the ultraviolet theory which appear to be unnatural. This is called the *naturalness* problem. In particular, if we accept that the Standard Model has an UV completion at the scale of the Planck mass, we would expect corrections proportional to $M_{\rm Pl}$. The huge discrepancy between the electroweak scale and the Planck mass is often referred to as the *hierarchy problem*.

- The cosmological constant is several order of magnitudes smaller than one would expects from the estimates done in the Standard Model, giving again a problem of naturalness.

- The Standard Model has a great number of free parameters: 19 without including the neutrinos masses and mixings (but including the θ parameter). In particular, there are 9 Yukawa couplings that describe the interaction strength of the fermions with the Higgs field. These display a hierarchical structure whose origin is not explained by the Standard Model.

From a phenomenological point of view, there are some experimental observations that have no explanation in the Standard Model:

- Neutrino oscillations, whose evidence is now well proven, require a mass different from zero for at least two flavours of neutrinos. It is still an open question what is the origin of their mass, that could be of the Majorana type (inducing lepton number violation), or Dirac (in which case it is necessary to introduce a new fermionic field, the right-handed neutrino, which is a singlet under the Standard Model gauge group).

- The QCD lagrangian includes a term $\theta G_{\mu\nu}\tilde{G}^{\mu\nu}$ which induces the violation of P and CP symmetries in the strong interactions. These effect would induce an electric dipole moment for the neutron, which is not observed. There is a strong upper bound $\theta < 10^{-10}$ and we would like to have an explanation for the occurrence of such a small number.

- Astrophysical observations ranging from galactic to cosmological scaled, suggest that the 80% of the matter present in the Universe in non-baryonic. The observations can be explained satisfactorily postulating the existence of a new form of matter which is usually referred to as *dark matter*.

1.2 The case for dark matter

Dark matter is an unknown form of non-baryonic matter whose existence is postulated in order to explain several astrophysical observations. Understanding the fundamental nature of dark matter is one of the most pressing unsolved problems in modern physics. At the moment, all the solid evidences for dark matter[2] are gravitational in nature. We shall briefly review the main observations and then present a summary of how they constrain the properties of dark matter.

1.2.1 Observational evidence

One of the most compelling evidences in favour of dark matter is that its existence can explain in a very economical and satisfactory way a great number of measurements on vastly different scales and based on much different techniques.

The first strong indication that a large fraction of the mass in the Universe is dark was provided by the measurement of the rotation curves of spiral galaxies [4, 5] (see also reference [6] for more recent results). By means of spectroscopic surveys it is possible to measure the circular velocity of the stars in a galaxy v_c as a function of their distance from the galactic center. From standard Newtonian mechanics one expects:

$$v_c = \sqrt{\frac{G_N M}{r}}$$

where M is the mass enclosed in a sphere of radius r, as dictated by the Gauss' law for classical gravity. For distances greater than the galaxy disk $r > R_{\text{disk}}$ the enclosed mass M should be constant (all the visible mass is concentrated in the disk); therefore we obtain the scaling law $v_c \propto r^{-1/2}$. Observations, however, give flat rotation curves at large distances (see figure 1.1), implying $M(r) \propto r$. This suggests that a vast portion of the matter in spiral galaxies is invisible and is not confined in the galactic disk. This component is called *dark matter*. From the accurate measurement of rotation curves it is possible to reconstruct the density profile of matter inside a galaxy [7]. The result, for spiral galaxies, is that while visible matter is distributed on a disk, dark matter form a spherical halo that extends on scales of order $R_{\text{halo}} \sim 100\,\text{kpc}$. These results are supported also by numerical studies on the stability of galaxies that show that spiral galaxies must be surrounded by an halo of dark matter [8]. Using the virial theorem it is possible to estimate also the average velocity of dark matter inside an halo, obtaining $v \sim 100\,\text{km/s}$; this tells us that dark matter must be non-relativistic.

Further evidence is provided by observations of clusters of galaxies on a typical scale of $R_{\text{cluster}} \sim 10\,\text{Mpc}$. Applying the virial theorem, knowing the velocity dispersion of the galaxies in a cluster, one can estimate the total mass contained in the cluster. Comparing this value with the mass of the observed galaxies, one obtains the ratio of dark and visible matter. This is found to be of order

$$\frac{M}{M_{\text{visible}}} \sim 100$$

[2]Leaving aside disputed claims of direct or indirect detection such as the anomaly observed by the DAMA/LIBRA experiment at Gran Sasso National Laboratory [3].

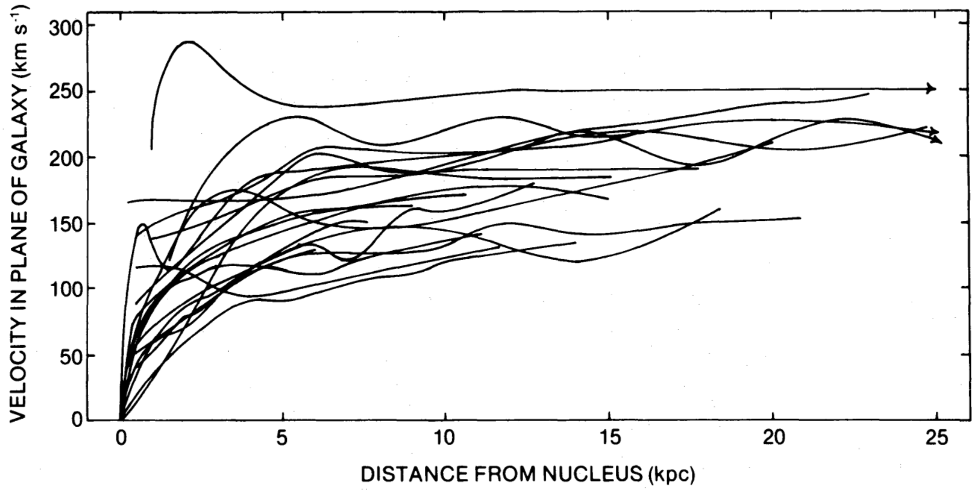


Figure 1.1: Rotation curves of spiral galaxies from the original article of Rubin et al. [4]. At large radial distances from the center, the curves appear to be flat, suggesting a distribution of matter $M(r) \propto r$. This is in contrast with the expectations based on the distribution of visible matter (*i.e.* stars).

in a typical cluster. Indeed, this was first done by the astronomer Fritz Zwicky in 1933 [9], who subsequently was the first to notice that some form of dark (*i.e.* invisible) matter was needed to explain the large ratio.

More recently, gravitational lensing observations have provided a new independent strong evidence for dark matter. A particular convincing observation has been the one concerning two clusters spotted just after their collision [10, 11]. The density profile reconstructed through lensing observation appears to be displaced with respect to the distribution of hot gas, obtained through X-ray observations. The hot gas component usually represents the 90% of the total budget of baryonic matter in a cluster, with galaxies representing the remaining 10%. Moreover the distribution of galaxies follows the gravitational potential, providing strong evidence that the cluster is dominated by dark matter (figure 1.2).

One of the first hypothesis advanced to explain the nature of dark matter was that it was composed by a population of compact astrophysical objects with low luminosity, such as planets, neutron stars and black holes (MACHO). However, gravitational microlensing surveys have excluded this possibility [12].

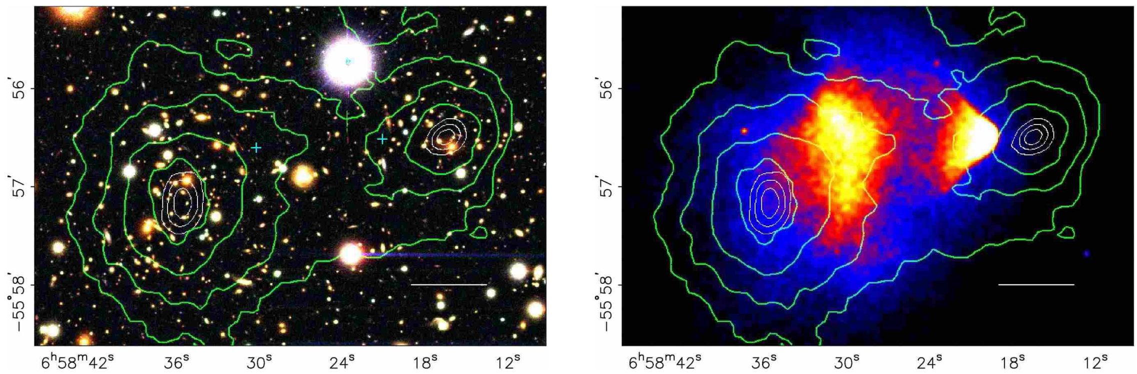
Finally, cosmological observations on scales $r > 100\,\mathrm{Mpc}$ strengthen the evidence for dark matter and clarify unequivocally its non-baryonic nature. Big Bang Nucleosynthesis [13] predicts the relative abundance of primordial elements in term of the parameter η, the ratio of the number of nucleons and photons

$$\eta = \frac{n_B}{n_\gamma}$$

In particular, astrophysical measurements [14] of the number ratio of deuterium to hydrogen give a baryon-to-photon ratio

$$\eta = (6.0 \pm 0.1)\, 10^{-10}$$

From this value and the knowledge of the number density of photons, which can be simply



(a) Superposition of a color image of the merging cluster 1E0657-558 with gravitational potential profile obtained through gravitational lensing observations (green lines).

(b) Superposition of an X-ray image of the merging cluster 1E0657-558 with gravitational potential profile obtained through gravitational lensing observations (green lines).

Figure 1.2: The distribution of hot plasma (signalled by X-ray emissions), which constitutes about the 90% of the baryonic matter in a cluster, is displaced with respect to the gravitational potential profile. This observation suggests that the cluster is dominated by dark matter.

obtained integrating the black body spectrum describing CMB radiation, it is possible to derive the present day baryon density parameter [14]

$$\Omega_B h^2 = 0.0220 \pm 0.0005$$

where h is the reduced Hubble constant defined by the relation $H_0 = 100h\,\mathrm{kms}^{-1}\ \mathrm{Mpc}^{-1}$. This value of Ω_b is much larger than the energy density of visible matter (*i.e.* stars) $\Omega_{\mathrm{lum}} h^2 \sim 0.0012$ [15], suggesting that a large portion of baryonic matter is given by intergalactic plasma. However, the obtained value cannot account for the cold matter density $\Omega_M h^2 \simeq 0.15$, which has been measured through studies of galaxy clusters and type-Ia supernovae [14]. This result gives a strong evidence that the vast majority of non-relativistic matter in the Universe is non-baryonic dark matter.

Yet another independent indication of the presence of dark matter comes from the precision measurement of the spectrum of anisotropies in the cosmic microwave background radiation (CMB). Fitting the spectrum observed by the Planck satellite, it has been possible to extract the values of the cosmological parameters [16], yielding to the values

$$\Omega_B h^2 = 0.02226 \pm 0.00023$$
$$\Omega_{DM} h^2 = 0.1186 \pm 0.0020$$

The agreement of the different independent determinations is impressive and strengthen the hypothesis that all the observations can be explained in terms of a new form of non-baryonic matter, whose elementary nature and properties are still largely unknown.

A caveat to this conclusion is represented by the scenario in which dark matter is in the form of primordial black holes, composed by ordinary matter and collapsed before the epoch of Big Bang Nucleosyntesis. This scenario has been tested through gravitational lensing observation

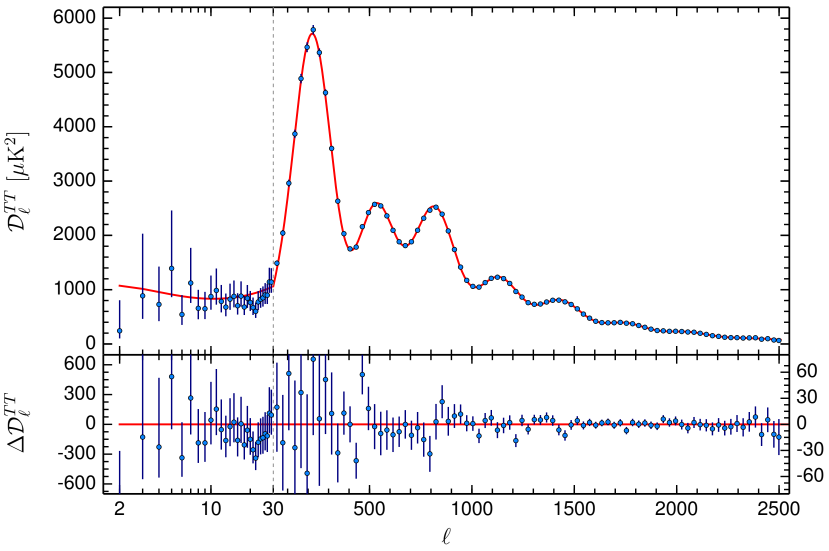


Figure 1.3: Temperature power spectrum of the cosmic microwave background radiation measured by Planck 2015 [16], compared with the best fit based on the ΛCDM cosmological model.

and almost all the parameter space has been ruled out [17].

Lastly, we stress that observations of bullet-like clusters provide a strong evidence in favour of the particle nature of dark matter [18]. Indeed, theories of modified gravity can explain in a simple way the observation of galactic rotation curves but encounter great difficulties in explaining the observed displacement among the gravitational potential and baryonic matter in the Bullet cluster. Further difficulties arise in explaining the cosmological observations, for which a fully relativistic theory is needed.

1.2.2 What we know about dark matter

A strong experimental and theoretical effort has been devoted to understand the nature of dark matter. Even though we did not uncover its nature and properties yet, we have gained some knowledge on the features that a dark matter candidate should satisfy in order to be phenomenologically viable.

In summary, what we know about dark matter is that:

- its relic abundance must be [16]: $\Omega_{DM}h^2 = 0.1186 \pm 0.0020$, as discussed in the previous section.

- it must be *cold*. Numerical simulations on the cosmological formation of large scale structures suggest the dark matter should be *cold*, meaning that it has to be non-relativistic already at the epoch of structure formation [19].

- it must be *non-baryonic*. As discussed in the previous section, Big Bang Nucleosynthesis and CMB data provide a determination of the density parameter for baryonic matter and dark matter separately, suggesting that dark matter is non-baryonic.

- dark matter self interaction cross section must satisfy the bound $\sigma/m \leq 1\,\mathrm{cm}^2\mathrm{g}^{-1}$. This has been obtained from astrophysical observations of bullet-like clusters [10, 11].
- a dark matter candidate must be neutral under the electromagnetic interactions [20].

Chapter 2

Composite dark matter

We want to study possible extensions of the Standard Model (SM) in which a dark matter candidate arises. Yet, we would like to keep the successful aspects of the SM. In this chapter we first analyse what are the properties of the SM that can explain its effectiveness and then try to use them as guidance principles (section 2.1).

We focus mainly on model building aspects and consider models with a *dark sector* with a non-Abelian gauge dynamics leading to accidentally stable composite dark matter candidates.

In section 2.2 we provide an overview of the framework we shall be considering and of the literature of works based on similar principles and motivations. The starting point of our work will be a critical analysis of the implicit assumptions of models of accidental composite dark matter based on vectorlike confinement (section 2.3).

Once we have determined what are these assumptions and why they render the models successful, we shall try to understand what are the possible generalisations. In section 2.4 we outline the possible routes that can be pursued to build new class of models for a composite dark sector with composite dark matter.

2.1 The successful paradigm of the Standard Model

The Standard Model (SM) lagrangian corresponds to the most general renormalizable lagrangian compatible with Lorentz and gauge invariance and comprising the SM field content. Its gauge theory structure, apart from being necessary in order to implement the massless spin 1 particles in the theory [21], is a fundamental ingredient in the proof of its renormalizability [22, 23].

The incredible effectiveness of the SM can be explained if we regard it as an effective field theory describing the *low energy* dynamics of a more fundamental UV theory, with an high cut-off $\Lambda_{\rm UV}$ [24]. The higher dimensional non-renormalizable operators are irrelevant in the infrared, being suppressed by powers of the cut-off scale.

$$\mathcal{L}_{eff} = \mathcal{L}_{\rm SM} + c^{(5)}\mathcal{O}^5 + \sum_i c_i^{(6)}\mathcal{O}_i^6 + \cdots$$

At low energies the dynamics is well described by the renormalizable lagrangian $\mathcal{L}_{\rm SM}$ which features some accidental global symmetries.

2.1.1 Accidental Stability

One of the great successes of the Standard Model is its ability to explain in a natural way the stability of all the matter that surrounds us in everyday life.

As we know, ordinary matter is composed by atoms, which in turn are bound states of nuclei (made of neutrons and protons) and electrons, held together by electromagnetic interactions mediated by the photon.

Any fundamental theory describing the Universe should explain the stability of ordinary matter in a satisfactory way. The Standard Model accomplish this task through the following explanations:

- The stability of the electron follows from charge and energy conservation: it is the lightest state charged under the electromagnetic interaction.
- The photon and graviton stability is a consequence of their vanishing mass and Lorentz invariance.
- Lorentz invariance ensures the stability also of the lightest neutrino, since it is the lightest spin 1/2 particle.
- The stability of the lightest baryon (the proton) is a consequence of an accidental symmetry, called *baryon number conservation.*

The stability of electron, neutrino, photon and graviton follows from deep physical principles: charge conservation, a consequence of the gauge structure of electromagnetic interactions, and Lorentz invariance. On the contrary, the stability of baryons seems somehow peculiar.

The baryon number conservation is a consequence of a global $\mathrm{U}(1)_{\mathrm{B}}$ symmetry of the Standard Model renormalizable lagrangian. At the non-renormalizable level, higher dimensional operators can induce a violation of this symmetry, resulting in the proton decay through processes such as $p \to e^+\pi^0$ or $p \to \nu\pi^+$. Baryon number violation arises at the level of dimension 6 operators, which are suppressed by two powers of the cut-off scale Λ_{UV}. Current bounds give $\tau(p \to e^+\pi^0) \geq 1.6 \times 10^{34}\,\mathrm{y}$ [25].

Indeed, already in the Standard Model the $\mathrm{U}(1)_{\mathrm{B}}$ symmetry is broken explicitly by quantum effects, since it has a non-vanishing anomaly with respect to the electroweak gauge interaction. However, the processes causing the baryon number violation involve some selection rules that prevent the proton from decaying [26]. In addition, the rate of baryon number violating processes is suppressed by non-perturbative factors, $\exp\left(-16\pi^2/g_2^2\right) \ll 1$, so that process involving baryon number violation have rates to small to be observationally relevant.

2.2 Gauge theories for composite Dark Matter

A dark matter candidate, in order to be successful needs to be stable on cosmological scales, *i.e.* its lifetime must be longer than the age of the Universe.

An appealing possibility is that the dark matter candidate is stable thanks to an accidental symmetry, similarly to what happens for the baryons in the Standard Model. In order for this to be the case, we restrict our attention to renormalizable models.

We focus here on this possibility and consider models in which there is a new non-abelian gauge group $\mathcal{G}_{\mathcal{DC}}$ inducing a strong dynamics and new fermionic fields (*dark quarks*). We refer to the field content added to the Standard Model as the *dark sector*. The composite states of the dark sector can be accidentally stable thanks to a global symmetry of the renormalizable lagrangian. If there is a neutral state which is accidentally stable, then it can be a successful dark matter candidate, provided it respects all the others phenomenological constraints and it reproduces the correct relic abundance.

We are interested in studying models in which the dark sector is coupled to the Standard Model sector through electroweak interactions and possibly Yukawa couplings, if allowed by the quantum numbers. Differently from technicolor and composite Higgs models, we require that the dark sector dynamics does not break the Standard Model gauge group, so that the new strong dynamics plays no role in electroweak symmetry breaking, which we describe through the ordinary Higgs mechanism. To stress the difference we shall refer to the new gauge interaction as *dark colour*.

We shall focus our attention on the dark matter candidate and on phenomenological bounds deriving from cosmological observations. However, we stress that these bounds are relevant also in the case in which the dark sector provides an extension of the Standard Model aimed to address an issue different from the nature of dark matter.

Schematically, the general framework we shall be considering is the following:

- A non-Abelian gauge theory with gauge group $\mathcal{G}_{\mathcal{DC}} = \mathrm{SU}(N)_{\mathrm{DC}}$ (*dark colour*)
- Fermions (*dark quarks*) charged under both $\mathcal{G}_{\mathcal{DC}}$ and $\mathcal{G}_{\mathcal{SM}}$
- No additional fundamental scalar fields
- Cancellation of the gauge anomalies in order to have a consistent and renormalizable quantum field theory
- Most general renormalizable lagrangian, including gauge interactions and Yukawa couplings with the Higgs field, if allowed by the quantum numbers

Models with composite dark matter have attracted some attention in the literature in the recent years and have been realised in a number of different contexts and with different phenomenological aims (representative example are the references [1, 27–30], for a review see reference [2]).

The vast majority of the models that have been proposed realise the framework of *vectorlike confinement* [31], that we shall briefly review in the next section. These models provide, more generally, a viable scenario for physics beyond the Standard Model and could be relevant also for phenomenological problems different from dark matter, such as flavour anomalies, unification or the strong CP problem.

2.3 Vectorlike confinement

Let us define some terminology. A representation r of a group G is said to be *real* if it is unitary equivalent to its complex conjugate, *i.e.* if its generators satisfy the relation

$$(iT^a)^* = SiT^aS^{-1}$$

or equivalently

$$T_a^T = -ST_aS^{-1} \tag{2.1}$$

where S is a unitary matrix. If $S^*S = 1$, or equivalently $S^T = S$, the representation is *strictly real* and it is possible to choose a base in which the generators T'_a are purely imaginary and antisymmetric matrices. If $S^*S = -1$, or equivalently $S^T = -S$, the representation is called *pseudo-real.* If the condition 2.1 is not satisfied, the representation is called *complex.*

Let us consider the fermion content organised such that all the fields are left-handed Weyl fermions.

We shall refer to a model as *real* if the fermions can be organised in a (reducible) representation of the group G which is real. This class of models includes the following subclasses:

- a model is *vectorlike* if for each irreducible representation there is a corresponding complex conjugate representation. This is equivalent to say that the model can be described in terms of Dirac fermions.
- a model is *strictly (pseudo) real* if all the irreducible representations are strictly (pseudo) real.

We shall refer to a model which is not real as *chiral* or *complex.*

Many of the models studied in the literature on composite dark matter realise the framework known as *vectorlike confinement* [31]. New fermions are introduced and assumed to be vectorlike under the dark gauge group, transforming in the fundamental (plus anti-fundamental) representation of $\mathcal{G_{DC}}$; correspondingly, they are assumed to be real under $\mathcal{G_{SM}}$ and globally real under $\mathcal{G_{DC}} \times \mathcal{G_{SM}}$. To be concrete, for $\mathcal{G_{DC}} = \mathrm{SU}(N)_{\mathrm{DC}}$, the (Weyl) fermion content is

$$\Psi = \bigoplus_{i=1}^{N_s} \left[(N, r_i) + \left(\bar{N}, \bar{r_i}\right)\right] \tag{2.2}$$

where N and $\bar{N}$ are the fundamental and anti-fundamental representations of $\mathrm{SU}(N)_{\mathrm{DC}}$ respectively and r_i is a generic irreducible representation of $\mathcal{G_{SM}}$. This is equivalent to say that the new fields are Dirac fermions transforming in the fundamental of $\mathcal{G_{DC}}$ and in the r_i representation of $\mathcal{G_{SM}}$.

Gauge anomaly cancellation is automatic in this scenario since the model is assumed to be real under $\mathcal{G_{DC}} \times \mathcal{G_{SM}}$ (see section 3.2 for a discussion).

The SM fields are singlet under $\mathcal{G_{DC}}$ and at the renormalizable level they interact with the dark sector only through gauge interactions and Yukawa couplings with the Higgs field - if allowed by the symmetries of the model, depending on the quantum numbers.

Since the dark sector is real under $\mathcal{G_{SM}} \times \mathcal{G_{DC}}$, explicit mass terms are allowed in the renormalizable lagrangian, without involving the Higgs sector. There are then N_s free mass parameters in addition to the dark colour scale $\Lambda_{\rm DC}$, where $2N_s$ is the number of distinct irreducible representations under $\mathcal{G_{SM}} \times \mathcal{G_{DC}}$, paired in couples to give Dirac fermions.

The fermion content is assumed to be such that the theory is asymptotically free and confining in the infrared, with confinement scale $\Lambda_{\rm DC}$. If the dark quarks have a mass lower than the confinement scale, this model corresponds to a QCD-like theory with $N_{\rm DC}$ colours and $N_f = \sum_{i=1}^{N_s} \dim(r_i)$ flavours, where the sum is restricted to the representations with mass $M_{\mathcal{Q}_\rangle} < \Lambda_{\rm DC}$.

There is then an approximate chiral symmetry, which becomes exact in the massless limit. At the classical level the flavour group is

$$\mathrm{G_F} = \mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R \times \mathrm{U}(1)_V \times \mathrm{U}(1)_A \tag{2.3}$$

However, the factor $\mathrm{U}(1)_A$ is anomalous under $\mathcal{G_{DC}}$ and is thus explicitly broken at the quantum level. Therefore we are left with

$$\mathrm{G_F} = \mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R \times \mathrm{U}(1)_V \tag{2.4}$$

The dynamics is then assumed to be analogous to QCD: chiral symmetry breaking occurs due to the formation of quark condensates; $\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R$ is broken to $\mathrm{SU}(N_f)_V$ and each broken generator is associated to a Goldstone boson. The chiral condensate associated to dark colour could in principle lead to a spontaneous breaking of the electroweak symmetry, in contrast with our requirement. However, the choice of real representations under $\mathcal{G_{SM}}$ ensures that there is always an orientation for the condensates such that they are electroweak singlets. This is expected to be dynamically preferred [32, 33].

If the flavour group was an exact symmetry group, the Goldstone bosons would be massless scalars; however, the explicit breaking of the flavour group induced by the dark quarks mass term induces a mass for these scalars, which in this case are usually called pseudo-Goldstone bosons. Furthermore, dark colour confinement ensures that asymptotic states are singlets of $\mathcal{G_{DC}}$ and thus composite hadronic states. As for QCD, these include mesons, baryons and glueballs.

The low energy dynamics can be efficiently described by means of chiral effective lagrangians and chiral perturbation theory techniques, valid in the regime $E \ll \Lambda_{\rm DC}$, similarly to what is done in Quantum Chromodynamics.

The lowest lying states are dark pions with a mass of order $M_\pi \approx \sqrt{M_Q \Lambda_{\rm DC}}$. Dark glueballs have a mass of order $M_\Phi \sim \Lambda_{\rm DC}$, while dark baryons mass scales with the number of dark colours $M_{\mathcal{B}} \approx N_{\rm DC}\Lambda_{\rm DC}$.

The unbroken flavour group

$$H_F = \mathrm{SU}(N_f)_V \times \mathrm{U}(1)_V$$

includes a $\mathrm{U}(1)_V$ factor which corresponds to dark baryon number conservation. This symmetry ensures the stability of the lightest dark baryon, which (if neutral) can be a valid dark matter candidate.

Summarising, the underlining assumptions of the models realising vectorlike confinement are:

- Vectorlike model under $\mathcal{G}_{\mathcal{DC}}$; dark quarks are Dirac fermions in the fundamental representation of $\mathcal{G}_{\mathcal{DC}}$;
- Real representation (vectorlike, strictly real or pseudo-real) under $\mathcal{G}_{\mathcal{SM}}$;
- Real representations under $\mathcal{G}_{\mathcal{DC}} \times \mathcal{G}_{\mathcal{SM}}$;
- Light dark quarks ($M_{\mathcal{Q}} < \Lambda_{\rm DC}$) and chiral symmetry breaking as in QCD;
- The dynamics is confining and the spectrum is similar to the one of QCD;
- The confinement scale is above the electroweak scale.

From these assumptions some of the properties we require for our dark sector follow directly:

- $\mathcal{G}_{\mathcal{DC}}$ anomaly cancellation;
- $\mathcal{G}_{\mathcal{SM}}$ anomaly cancellation;
- Dynamics does not break $\mathcal{G}_{\mathcal{SM}}$;
- Mass terms arise prior to electroweak symmetry breaking;
- Accidental symmetries ensure the stability of dark baryons.

2.3.1 Accidental Composite Dark Matter

The phenomenological implications of composite dark matter models based on the framework of vectorlike confinement have been explored in [34].

Under the following two additional assumptions:

- partial SU(5) unification;
- the SM gauge couplings have no Landau poles below M_{Pl};

the authors provide a complete list of dark matter models and study their phenomenology.

If the relic density is produced thermally (through a mechanism that we shall review in chapter 5), in order to reproduce the dark matter relic abundance dark baryons should have a masses of order $M_{\rm DM} \sim 100\,\text{TeV}$. We stress that even though the dark matter candidate is weakly interacting, *i.e.* it is the neutral component of an electroweak multiplet, it is not a standard WIMP. Indeed, its relic density is not set by the rate of annihilation in Standard Model particles through electroweak interactions but by the annihilation rate in dark pions, which in turn decay in Standard Model particles. Since this process is mediated by dark color interactions, the cross section is much larger, requiring an higher mass for the dark matter candidates.

Altogether, these models provide a well motivated and rich phenomenology for a dark sector. The main features can be summarised as follows:

- Rich phenomenology at scales accessible by present or near future experiments (collider, direct, indirect);
- Anomaly cancellation: extension of the fermionic content in a coherent way;
- Flavour and EW precision tests limits avoided in a simple way;
- Composite dark matter candidates stable thanks to accidental symmetries, similarly to what happens in the Standard Model for the proton;
- May improve coupling constants unification.

2.4 Beyond vectorlike confinement

Models realising vectorlike confinement represent an interesting possibility for a composite dark sector. However they are based on a quite restrictive set of assumptions, and represent a small subset of the possible gauge theories. We want to understand what are the other classes of gauge theories with a different phenomenology that could be viable models for a composite dark sector.

Relaxing this assumption we consider models in which the field content is given by Weyl fermions transforming under generic irreducible representations of $\mathrm{SU}(N)_{\mathrm{DC}}$. To be concrete, we shall focus our attention on models built using representations with up to two indices.

The models need not to be necessarily vectorlike nor real. An interesting possibility is that of chiral models, which we shall analyse in more detail in chapter 3. If this is the case, however, the cancellation of gauge anomalies is no longer ensured. In order to have a consistent theory we must impose this further constraint that gives non-trivial restrictions on the possible choices of representations of the gauge group.

Moreover, we require that the infrared dynamics does not break the Standard Model gauge group. Indeed, the formation of condensate could induce a dynamical breaking of the Standard Model gauge group $\mathcal{G}_{\mathcal{SM}}$, producing a technicolor model. We are interested in the opposite case and this requirement gives again a non-trivial condition.

Lastly, we must check that the physical spectrum and the phenomenology are compatible with the experimental observations.

Summarising, we shall work under the following assumptions:

- Weyl spinors in up to two indices irreducible representations of $\mathrm{SU}(N)_{\mathrm{DC}}$ and similarly for $\mathcal{G}_{\mathcal{SM}}$;
- Models not necessarily vectorlike;
- UV: $\mathcal{G}_{\mathcal{DC}}$ anomaly free;
- UV: $\mathcal{G}_{\mathcal{SM}}$ anomaly free;
- UV: Renormalizable lagrangian;
- UV: No Landau poles at low energy for the gauge couplings;

- IR: the dark sector dynamics does not break $\mathcal{G}_{\mathcal{SM}}$;
- IR: There are no massless or ultra-light states in conflict with experimental observations.

We can identify two broad categories of models realising a dark sector with composite dark matter, depending on whether the strong dynamics induces the formation of fermionic condensates (and thus the spontaneous breaking of the global symmetry group) or not.

The first class comprises the familiar case of models realising vectorlike confinement and, more generally, of models with fermions transforming as real representations under $\mathcal{G}_{\mathcal{DC}} \times \mathcal{G}_{\mathcal{SM}}$ (not necessarily vectorlike), which have a dynamics qualitatively similar to the one discussed in the previous section.

Models with a complex (reducible) representation under $\mathcal{G}_{\mathcal{DC}} \times \mathcal{G}_{\mathcal{SM}}$ (*i.e. chiral models*) and no additional fundamental scalar fields are expected to break dynamically the gauge group, through the formation of condensates. This scenario of is somehow peculiar, and can feature a dynamics much different than the one of models with vectorlike confinement. Chiral models arise naturally in the context of Grand Unified Theories, and the Standard model itself is a chiral gauge theory. We shall discuss the motivations for considering this class of models as a dark sector and some aspects of their dynamics in chapter 3.

The second class includes models with dark quarks heavier then the confinement scale, that have been recently considered [35], and models in which the dark colour dynamics has an infrared fixed point. Models with infrared fixed points have been considered in the context of technicolour models that try to explain the dynamics of the electroweak symmetry breaking [36]. In this work we shall analyse the possibility that a dark sector is a gauge theory with an infrared conformal dynamics, broken by the presence of dark quarks mass terms. Differently from technicolor we consider the case in which the dynamics does not break the Standard Model gauge group. We analyse this scenario in chapters 4 and 5, both from the point of view of model building and phenomenology.

Chapter 3

Chiral Models

We want to understand if a model with chiral field content can be a valid framework for a dark sector. We restrict our attention to dark sectors with a simple gauge group, and to unitary gauge groups $\mathrm{SU}(N)_{\mathrm{DC}}$ in particular.

In section 3.1 we give an overview of the general properties of chiral models and discuss some additional motivations that explain why this class of models could be interesting. Subsequently, in 3.2, we review the concept of anomaly and summarise the conditions that a gauge theory must satisfy in order to be anomaly free and thus consistent. 't Hooft anomaly matching is discussed and exemplified through applications to the Standard Model, that give a different perspective on some well-known facts. In section 3.3 we analyse the dynamics of models with a chiral field content under $\mathcal{G}_{\mathcal{DC}}$ and discuss some model building issues on what are the necessary conditions in order to have a model that does not break $\mathcal{G}_{\mathcal{SM}}$ dynamically. In the last section (3.4) we present a general argument based on 't Hooft anomaly matching that explains why, and under which assumptions, chiral models have massless asymptotic states in the infrared. We then provide a brief overview of some of the mechanisms that can induce a mass term for the would-be massless particles.

3.1 General properties

A chiral model a model in which fermionic fields[1] transform in a complex (reducible) representation of the gauge group $\mathcal{G}_{\mathcal{DC}} \times \mathcal{G}_{\mathcal{SM}}$. In physical terms this translates in the condition that the fermions cannot have explicit mass terms.

An appealing feature of chiral dark sectors is that, in absence of elementary scalar fields charged under dark colour, they are characterised by a unique energy scale, Λ_{DC}, which is generated dynamically through dimensional transmutation.

In this scenario, the masses of all the asymptotic states in the dark sector arise dynamically from the scale Λ_{DC} induced by the dark gauge interactions; furthermore they are naturally small with respect to the cut-off of the theory (*e.g.* the Planck scale). The small value of the mass is

[1] We describe our theory in terms of left-handed fermionic fields only; right-handed fields can be obtained by charge conjugation $\psi_R = i\sigma_2\psi_L^*$.

not only technically natural[2] but a consequence of the dynamics.

Indeed, if all the dark quarks transform in complex representations there are no explicit mass terms. Therefore, all the masses of the asymptotic states in the dark sector arise dynamically from the scale Λ_{DC} induced by the dark gauge interactions. This is an attractive scenario since there would be a single dynamical mass scale, with the masses of the states in the dark sector all related among each other and dark matter stability ensured by accidental symmetries of the renormalizable gauge theory.

Chiral models play a central role in high energy physics. A well-known example of a chiral gauge theory is the Standard Model itself: the fermions have no explicit mass term in the lagrangian and acquire a mass only through the Yukawa interaction with the Higgs field, after electroweak symmetry breaking. Also in this case, the mass of the fermions is generated dynamically and is of the order of the Higgs vacuum expectation value or smaller, leading to naturally light particles.

In the context of grand unified theories, chiral models play a central role. Indeed, if the Standard Model gauge group is unified to a simple gauge group at an high energies, the model must be a chiral theory. This is just what happens in the case of Georgi-Glashow SU(5) grand unification theory [37] in which the standard model fermions are organised in two irreducible representations: an antisymmetric 10 and an anti-fundamental $\bar{5}$, each one coming in three families.

It is then natural to explore the possibility that the dark sector is a chiral theory. This alternative has not received much attention in the literature on dark sectors and can lead to a phenomenology vastly different from the one of models based on real representations.

Models with a non-Abelian gauge group $\mathrm{SU}(N)$ with chiral representations were studied in the context of technicolor theories [38, 39], where they were proposed as a way to generate dynamically a hierarchy of mass scales for quarks and leptons.

As for dark sector model building, chiral models have been considered recently by a few authors. The case of an abelian dark sector with a chiral U(1) broken through the Higgs mechanism by charged scalars has been considered in [40–43].

Recently, Nomura et al. have considered a model with a strongly interacting chiral dark sector [44, 45]. Their model is based on a non-simple gauge group $\mathrm{SU}(N)_{\mathrm{DC}} \times \mathrm{U}(1)$ with fermions transforming in representations real under $\mathrm{SU}(N)_{\mathrm{DC}}$ and U(1) but complex under the product; no fundamental scalar fields are introduced and the abelian U(1) factor is broken dynamically by the dark sector. The dark fermions are singlets under the Standard Model, and the two sectors are coupled only through vector boson mixing of the Z' with hypercharge.

Chiral dark sectors have been considered, with a different perspective, also in the context of models with a mirror world [46, 47]. In these models the dark sector is a copy of the Standard Model with the same particle content and the two sectors can communicate gravitationally and through the Higgs portal. Mirror baryons, stable thanks to the accidental baryon number conservation, can account for dark matter.

[2] A parameter with a small value is said to be technically natural if in the limit in which it vanishes the model acquires an enhanced symmetry.

In this work we study the following alternative, that has not been considered yet in the literature: a chiral dark sector with non-Abelian gauge group, featuring a strongly interacting dynamics in the infrared, and with fermions charged under both $\mathcal{G}_{\mathcal{DC}}$ and $\mathcal{G}_{\mathcal{SM}}$.

Chiral models can be classified in four different classes, according to the reality properties of the representations under $\mathcal{G}_{\mathcal{DC}}$ and $\mathcal{G}_{\mathcal{SM}}$.

Table 3.1: SU(N) irreps with up to two indices.

	$\mathcal{G}_{\mathcal{DC}}$	$\mathcal{G}_{\mathcal{SM}}$	$\mathcal{G}_{\mathcal{DC}} \times \mathcal{G}_{\mathcal{SM}}$
1)	Real	Real	Complex
2)	Real	Complex	Complex
3)	Complex	Real	Complex
4)	Complex	Complex	Complex

We shall focus on models in which the fermions in the dark sector are charged under the Standard Model but the dark gauge dynamics dominates, *i.e.* the dark sector becomes strongly coupled at a scale $\Lambda_{\rm DC} \gg m_h$. We do so because we are interested in understanding if there are viable alternatives for dark sectors with fermions interacting with the Standard Model. The dark sector dynamics is expected to induce a mass for the composite states of order $\Lambda_{\rm DC}$ (unless they are Goldstone bosons or unconfined fermions). If we require $\Lambda_{\rm DC} \gg m_h$, the confined composite resonances (in particular the ones charged under $\mathcal{G}_{\mathcal{SM}}$) will be heavier than the electroweak scale, explaining why they have not been observed yet.

In this scenario, the formation of condensates of dark fermions due to the gauge dynamics could induce a dynamical breaking of the electroweak symmetry (as in technicolor models). We focus on the case in which the $\mathcal{G}_{\mathcal{SM}}$ is left unbroken by the dark sector dynamics, *i.e.* they are not *technicolor* theories, but theories describing a dark sector.

As we discuss in the next paragraph, models realising the scenarios 1) and 2) are incompatible with this last request; the rest of the chapter will focus on models with complex representations under $\mathcal{G}_{\mathcal{DC}}$, realising the scenarios 3) and 4).

Models with fermions real under $\mathcal{G}_{\mathcal{DC}}$

Models in the first two classes, *i.e.* real under $\mathcal{G}_{\mathcal{DC}}$ but chiral under $\mathcal{G}_{\mathcal{SM}}$ or the product $\mathcal{G}_{\mathcal{DC}} \times \mathcal{G}_{\mathcal{SM}}$, cannot be compatible with the requirement that the strong dynamics of $\mathcal{G}_{\mathcal{DC}}$ does not induces a dynamical breaking of $\mathcal{G}_{\mathcal{SM}}$.

Let us first review what are the possible patterns of chiral symmetry for $\mathrm{SU}(N)$ gauge theories with different representations.

For vectorlike models with N_f Dirac fermions transforming in a complex representation, the global symmetry group is $\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R$. The pattern of chiral symmetry breaking, analogous to the one realised in QCD, is

$$\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R \times \mathrm{U}(1)_V \longrightarrow \mathrm{SU}(N_f)_V \times \mathrm{U}(1)_V$$

This pattern of symmetry breaking describes extremely well the observations of hadronic physics, providing a powerful principle to classify mesons and baryons. It has been proven theoretically [48] that the vectorial subgroup $\mathrm{SU}(N_f)_V \times \mathrm{U}(1)_V$ cannot be broken spontaneously, and since this pattern is the one that leaves unbroken the largest possible subgroup, one can safely conclude that it is the only possible pattern of symmetry breaking in vectorlike theories. This has been confirmed by lattice calculations.

Let us now consider the case of models with real representations. We consider a model with $2N$ massless Weyl fermions, transforming as a real representation. In this case the global symmetry is enhanced to $\mathrm{SU}(2N)$. The reality condition for the representation can be expressed as

$$S^{-1}(iT^a)S = (iT^a)^*$$

where S is a unitary matrix. For a *strictly real* representation S is symmetric, *i.e.* $S^T = S$, while for a *pseudoreal* representation S is antisymmetric, *i.e.* $S^T = -S$. Exponentiating this relation we have

$$S^{-1}US = U^*$$

and using the unitarity of U and S we obtain

$$U^T S^{-1} U = S^{-1}$$

We want to find a matrix Σ such that the condensate

$$\langle \psi^T \Sigma \psi \rangle \neq 0$$

is a singlet of the gauge group. Choosing $\Sigma = S^{-1}$ we see that this condition is automatically satisfied. Since the inverse of a (anti)symmetric matrix is (anti)symmetric, from the properties of S we conclude that

- Σ is symmetric for a strictly real representation;
- Σ is antisymmetric for a pseudoreal representation;

Assuming that the dynamics leaves unbroken the maximal flavour subgroup [32, 33], we obtain that:

- for a model with $2N$ Weyl fermions transforming in a strictly real representation the condensate is Σ is proportional to the identity and the pattern of symmetry breaking is

$$\mathrm{SU}(2N) \longrightarrow \mathrm{SO}(2N)$$

- for a model with $2N$ Weyl fermions transforming in a pseudoreal representation the condensate is Σ is a symplectic matrix and the pattern of symmetry breaking is

$$\mathrm{SU}(2N) \longrightarrow \mathrm{Sp}(2N)$$

For pseudoreal representations the matrix Σ has necessarily even dimension, while for strictly real representations it is possible to extend the conclusion to system with an arbitrary number of Weyl fermions N_w.

A generalisation of the Vafa-Witten theorem has been proven by Kosower [49], showing that the unbroken group suggested by the maximal flavour subgroup criterion cannot be broken spontaneously. Lattice simulations of models with real representations have confirmed these symmetry breaking patterns [50, 51].

Having analysed what is the pattern of flavour symmetry breaking for models with real representations, we can now prove the following lemma: a model with field content real under $\mathcal{G}_{\mathcal{DC}}$ but chiral under $\mathcal{G}_{\mathcal{DC}} \times \mathcal{G}_{\mathcal{SM}}$ and $\Lambda_{\rm DC} \gg m_H$ leads to dynamical breaking of $\mathcal{G}_{\mathcal{SM}}$.

Indeed, since the dark colour dynamics dominates, dynamical breaking of the (approximate) flavour symmetry group occurs, realising one of the patterns described above (*i.e.* the condensate involves all the dark quarks). Being a singlet under $\mathcal{G}_{\mathcal{DC}}$, the condensate cannot be a singlet of $\mathcal{G}_{\mathcal{SM}}$ otherwise all the dark quarks could have an explicit mass term in the renormalizable lagrangian and the theory would not be chiral.

3.2 Anomalies

Before dealing with the dynamics of a chiral model, we shall review the concept of anomaly and some of its applications. We shall use the results of this section in our analysis of chiral dark sectors.

An anomaly is an obstruction to the realisation of a classical symmetry at the quantum level: one of the signals of the quantum violation of the symmetry is the appearance of anomalous terms in the Ward-Takahashi identities.

Anomalies provide a tool that can be used to make concrete prediction on the non-perturbative dynamics of quantum field theories.

First of all, the presence of an anomaly in a gauge theory signals an inconsistency. In the case of chiral gauge theories, the request of gauge anomaly cancellation induces strong constraints on the structure of the model, that must be satisfied in order to have a consistent theory. For instance, the cancellation of gauge anomalies in the Standard Model provides a strong consistency check.

Moreover, anomalies are an infrared effect, in the sense that only massless particles can contribute to the anomaly. As a consequence, they can be used to obtain information on the infrared structure of the gauge theory. In particular, through the so called *anomaly matching*, it is possible to predict some of the properties of the low energy effective theory describing the model in the infrared, even if the dynamics is strongly interacting and non-perturbative.

Finally, they are of great importance also from the phenomenological point of view: in QCD they explain the decay of the neutral pion into two photons, predicting a lifetime in excellent agreement with the experimental observations.

Let us consider a field theory described by an action $\mathcal{S}_{cl}$ and assume that there is a continuous symmetry group G that leaves the classical action invariant: $\delta\mathcal{S}_{cl} = 0$.

By Noether theorem there is a conserved current $j_\mu(x)$ which satisfies, at the classical level, the continuity equation $\partial_\mu j^\mu = 0$. This equation expresses a local conservation law of the classical system. At the global level, if the symmetry is linearly realised and if the fields are localised, this implies the conservation of the charge associated to this current[3]

$$Q = \int d^3x j^0(\vec{x}, t) \qquad\qquad \frac{\mathrm{d}}{\mathrm{d}t} Q(t) = 0$$

The symmetry G is said to be anomalous if it is violated at the quantum level. If there is an anomaly, the quantum analogue of the continuity equation[4], $\langle \partial_\mu j^\mu \rangle = 0$, receives quantum corrections, giving rise to *anomalous Ward identities* such as

$$\langle \partial_\mu j^\mu \rangle = \mathcal{A}$$

where $\mathcal{A}$ is referred to as the *anomaly.*

If G represents a global symmetry group, then the anomaly results simply in an anomalous non conservation at the quantum level of the Noether current associated to the symmetry.

On the other hand, if G is a gauge group, the presence of an anomaly would indicate an inconsistency of the theory. Indeed, a gauge invariance is a redundancy in the degrees of freedom we our using to describe our system. The physical degrees of freedom are in the quotient of the space of fields by gauge transformations, *i.e.* gauge invariance is required to remove unphysical degrees of freedom. Therefore, in order to have a consistent gauge theory, it is necessary to require the cancellation of gauge anomalies.

3.2.1 Chiral anomaly

To be concrete, let us consider QED in four dimensions with one massless Dirac fermion ψ with unit charge, described by the lagrangian

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\bar{\psi} D_\mu \gamma^\mu \psi$$

We focus our attention to the case in which there is a dynamical fermion in a background with fixed electromagnetic fields. In the path integral formulation all the correlation functions can be computed by taking derivatives of the generating functional

$$Z[\eta, \bar{\eta}] = \int D\psi D\bar{\psi} e^{i\mathcal{S} + i\int \bar{\eta}\psi + i\int \bar{\psi}\eta}$$

Let us consider axial transformations $\psi \to e^{i\alpha\gamma_5}\psi$. This is a global symmetry group of the classical action since it leaves unchanged the lagrangian. Considering a local transformation

[3]Otherwise, if the symmetry in non-linearly realised (*i.e.* there is spontaneous symmetry breaking) the charge Q is not a well defined quantity and it is not a good quantum number.

[4]We indicate as $\langle \cdot \rangle$ the expectation value on the vacuum of the operator in bracket.

associated to the group we have

$$\delta\mathcal{S} = \int d^4x(\partial_\mu\alpha(x))j_5^\mu(x)$$

where $j_5^\mu(x) = \bar{\psi}\gamma^\mu\gamma^5\psi$ is the Noether current associated to the classical symmetry. Defining carefully the integration measure in the path integral, following the approach of Fujikawa [52], one finds that in presence of massless fermion the integration measure is not invariant under the axial transformation:

$$D\psi D\bar{\psi} \longrightarrow e^{i\int d^4x\alpha(x)\mathcal{A}(x)}D\psi D\bar{\psi}$$

where

$$\mathcal{A}(x) = -\frac{e^2}{16\pi^2}\varepsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}$$

The path integral transforms as

$$\int D\psi D\bar{\psi}e^{i\mathcal{S}} \longrightarrow \int D\psi D\bar{\psi}e^{i\int\alpha\mathcal{A}}e^{i\mathcal{S}+i\delta\mathcal{S}} \simeq \int D\psi D\bar{\psi}e^{i\mathcal{S}}\left[1 + \int d^4x(\alpha(x)\mathcal{A}(x) + (\partial_\mu\alpha(x))j_5^\mu(x))\right]$$

Integrating by parts and requiring the invariance of the path integral under change of variables, for an arbitrary parameter $\alpha(x)$, we arrive at

$$\langle\partial_\mu j_5^\mu\rangle_A = -\frac{e^2}{16\pi^2}\varepsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma} \tag{3.1}$$

where we are working with a fixed background field $A^\mu(x)$ and dynamical fermions, so that the expectation value is given by

$$\langle\partial_\mu\mathcal{O}\rangle_A = \frac{\int D\psi D\bar{\psi}\ \mathcal{O}\ e^{i\mathcal{S}[\psi,A]}}{\int D\psi D\bar{\psi}\ e^{i\mathcal{S}[\psi,A]}}$$

The anomaly can be computed directly by means of perturbation theory evaluating the triangle diagram with a loop of fermions, two external gauge bosons and an axial current insertion, as was first done by Adler [53], and Bell, Jackiw [54].

From a more rigorous point of view, the anomaly can be seen as a consequence of the regularisation procedure [55]. The current $j_5^\mu = \bar{\psi}(\vec{x},t)\gamma^\mu\gamma^5\psi(\vec{x},t)$ is ill defined since it involves the product of two operators at the same point which in general is divergent and needs some kind of regularisation. It turns out that there is no regularisation procedure consistent both with $\mathrm{U}(1)_A$ and $\mathrm{U}(1)_V$ symmetries; preserving $\mathrm{U}(1)_V$ gauge invariance gives an anomaly in the axial current. Through a point splitting regularisation one can define the current as

$$j_5^\mu = \lim_{\varepsilon\to 0}\bar{\psi}(x+\varepsilon/2)\gamma^\mu\gamma^5 e^{-ie\int_{x-\varepsilon/2}^{x+\varepsilon/2}\mathrm{Adx}}\psi(x-\varepsilon/2)$$

Computing the divergence and using the Dirac equation, then taking the limit $\varepsilon \to 0$, one obtains again equation 3.1.

Generalising upon the previous example, one can consider a system of massless fermions coupled to a non-abelian gauge theory. For a non-abelian chiral current

$$j^{\mu}_{5,i} = \bar{\psi}\gamma^{\mu}\gamma^{5}\tau_i\psi$$

one obtains the anomaly [21]:

$$\partial_\mu j^{\mu}_{5,i} = -\frac{g^2}{16\pi^2}\varepsilon_{\mu\nu\rho\sigma}\,\mathrm{Tr}\,[\tau_i T_a T_b] G^{\mu\nu}_a G^{\rho\sigma}_b \tag{3.2}$$

Gravitational anomaly

Similar methods can be used to compute the anomaly for the axial current once the fermions are coupled to gravity [56]. In particular, the fermion triangle with one axial current and two energy momentum tensors at the vertices has an anomaly

$$D_\mu j^{\mu}_{5} = -\frac{1}{384\pi^2}\frac{1}{2}\varepsilon_{\mu\nu\rho\sigma} R^{\mu\nu}{}_{\alpha\beta} R^{\rho\sigma\alpha\beta} \tag{3.3}$$

For a non-abelian chiral current

$$j^{\mu}_{5,i} = \bar{\psi}\gamma^{\mu}\gamma^{5}\tau_i\psi$$

the gravitational anomaly is proportional to $\mathrm{Tr}\,[\tau_i]$, which vanishes for currents associated to $\mathrm{SU}(N)$ global groups.

3.2.2 Gauge anomaly cancellation

So far we have discussed the anomalous non conservation of a global axial current in the presence of external background gauge fields coupled to vectorial currents. In chiral models we deal with a situation in which the gauge currents are themselves chiral.

For a chiral model we express the lagrangian in terms of left-handed fermionic fields ψ_{Li} in a representation r of the gauge group

$$\mathcal{L} = \psi^{\dagger}_{Li} i\bar{\sigma}_\mu D^{\mu}_{ij}\psi_{Lj} + \cdots$$

with spinor indices suppressed and covariant derivative defined as

$$D^{\mu}_{ij} = \partial^{\mu}\delta_{ij} - igA^{\mu a}\left(T^{a}_{(r)}\right)_{ij}$$

Anomaly $\mathrm{SU}(N)^3$

The anomaly in the divergence of the gauge current is given by the triangle diagram with the current at one vertex and the gauge bosons at the other two vertices [21]. It is proportional to

$$\mathrm{Tr}\left[T^{a}_{(r)}\left\{T^{b}_{(r)}, T^{c}_{(r)}\right\}\right]$$

If the representation r is reducible one obtains a sum on the irreducible representations.

For an SU(N) gauge theory there is a unique totally symmetric three index tensor up to a constant depending on the irreducible representation, known as the *anomaly coefficient*

$$\mathrm{Tr}\left[T^a_{(r_i)}\left\{T^b_{(r_i)}, T^c_{(r_i)}\right\}\right] = A(r_i)d^{abc} \tag{3.4}$$

If a representation is equivalent to its complex conjugate (*i.e.* it is real or pseudoreal), the anomaly coefficient vanishes automatically. Indeed one has

$$\left(iT^a_{(r_i)}\right)^* = S\left(iT^a_{(r_i)}\right)S^{-1} \Longrightarrow \left(T^a_{(r_i)}\right)^T = -S\left(T^a_{(r_i)}\right)S^{-1}$$

from which it follows that $A(r_i)d^{abc} = -A(r_i)d^{abc}$ and so $A(r_i)d^{abc} = 0$.

Some gauge groups such as SU(2) have only real or pseudoreal representations, therefore the coefficient d^{abc} is identically zero and there is no anomaly.

For SU(N) gauge groups with $N \geq 3$, in order to have a consistent gauge theory it is necessary to have a cancellation between the anomalies of the different representations, therefore we obtain the condition

$$\sum_i A(r_i) = 0$$

Since the anomaly in the current is caused by the gauge bosons to which the current itself is coupled, this anomaly is referred to as the SU(N)^3 anomaly.

SU(2) global gauge anomaly

Gauge anomalies are called local if they are associated with elements connected to the identity and global if they are associated to gauge transformation that cannot be transformed continuously into the identity (so called *large* gauge transformations).

Witten has shown in [57] that theories based on a gauge group with non-trivial fourth homotopy group have an additional consistency condition arising from the transformation properties of the generating functional Z under large gauge transformations.

As a consequence, an SU(2) gauge theory[5] must have an even number of doublets in order to be consistent. More generally it must have an even number of fermions transforming as SU(2) representations of even dimension, while the number of odd dimension representations can be arbitrary.

No further constraint arises for SU(N) gauge theories for $N \geq 3$, nor for O(N) gauge theories with $N \geq 6$ since they have trivial fourth homotopy group. Non trivial conditions arise only for Sp(N) gauge theories which have $\pi_4(\mathrm{Sp}(N)) = \mathbb{Z}_2$ [57].

Mixed gauge and gravitational anomalies

If the gauge group is not a simple group, mixed anomalies, with one gauge boson at one vertex and two background external gauge bosons (associated to the other factor of the gauge group), must be taken into account.

[5] The fourth homotopy group of SU(2) is $\pi_4(\mathrm{SU}(2)) = \mathbb{Z}_2$ [57].

The mixed anomalies $\mathrm{SU}(N)\,\mathrm{SU}(M)^2$ and $\mathrm{SU}(N)\,\mathrm{U}(1)^2$ are automatically zero since they are proportional to the trace of the generators of $\mathrm{SU}(N)$, which vanishes identically.

For $\mathrm{U}(1)\,\mathrm{SU}(N)^2$ one has the condition

$$\mathrm{Tr}\,[QT_aT_b]$$

where Q is the charge operator regarded as a matrix acting on left-handed fermionic fields. We note that only fermions charged under both the gauge groups contribute to the anomaly.

Similarly, coupling our model to gravity, for consistency we must include the cancellation of the gravitational anomalies $\mathrm{SU}(N)\mathrm{grav}^2$ and $\mathrm{U}(1)\mathrm{grav}^2$ [56]. The first vanishes identically as described before, while the second gives the condition

$$\mathrm{Tr}\,Q = 0$$

where the trace is on left-handed massless fermions.

3.2.3 't Hooft anomaly matching

The following argument has been put forward by 't Hooft in [58] and uses anomalies to gain information on the non-perturbative dynamics of a strongly interacting gauge theory.

Let us consider an asymptotically free $\mathrm{SU}(N)$ gauge theory with (massless) chiral fermions and a global symmetry group G, free of mixed anomalies $G \cdot \mathrm{SU}(N)^2$ (*i.e.* G is valid even in the presence of background gauge fields). Assume that there is an anomaly G^3, which in the ultraviolet picture can be computed through a triangle diagram with massless elementary fermions running in the loop. In the infrared, whatever the strong dynamics gives, the anomaly must be exactly reproduced if computed in terms of the massless particles of the exact physical spectrum (which can be either fermions or Goldstone bosons).

This can be understood in the following way. Imagine that we introduce a set of massless *spectator* fermions that are gauge singlets but contribute to the G^3 anomaly compensating exactly the existing anomaly. Now the group G is completely free of anomalies and we can gauge it, with a *weak* coupling so that the original interaction dominates the dynamics. Running in the infrared, the original theory flows towards a strong coupling regime (in QCD, for instance, the theory confines at low energies) and the original massless fermions can be replaced by a different set of asymptotic states (baryons and mesons in QCD). The spectator fermions, on the contrary, are gauge singlets and do not take part to the strong dynamics; therefore, they give the same G^3 anomaly as before. Since we started with a consistent theory in the ultraviolet, the effective low energy theory must be consistent too and so there must be massless states in the strong sector that cancel again the G^3 anomaly.

The argument does not depend on the value of the gauge coupling constant of the group G (provided it is *weak*), therefore we can take it to zero. The spectator fermions are now completely decoupled and the conclusion on the existence of massless states must be true also in the theory in which G is a global group and there are no spectator fermions.

In the low energy effective theory the anomaly can be reproduced by massless fermions (composite fermions if the theory is confining) or Goldstone bosons. In particular, in the effective theory describing the Goldstone bosons the anomaly is reproduced by the Wess-Zumino-Witten term [59, 60].

The core of the argument resides in the fact that we can add spectator fermions that do not interact under the strong dynamics but carry an anomaly under G^3, in such a way to cancel the existing flavour anomaly. The same line of reasoning can be applied to the gravitational anomaly $G \cdot \mathrm{grav}^2$, since the spectator fermions are coupled gravity, as was pointed out in [56].

This observation results in a second, independent, consistency condition: for any conserved global charge Q, the trace $\mathrm{Tr}\,[Q]$ must be equal if computed on the elementary left-handed fermions or on the fermions of the physical spectrum, unless the corresponding $\mathrm{U}(1)$ symmetry is spontaneously broken. Since only massless fermions can contribute to the anomaly, a non-zero value for $\mathrm{Tr}\,[Q]$ implies the existence of massless states, either fermions or Goldstone bosons from the spontaneous breaking of the symmetry.

A rigorous derivation of t'Hooft anomaly matching from general principles (analyticity and unitarity) has been provided by Coleman and Grossman [61].

Massless QCD example

Originally, the anomaly matching was used to constrain the infrared dynamics of QCD, specifically to show that spontaneous chiral symmetry breaking is unavoidable.

Let us consider QCD with three flavours of massless quarks (we are working in the so called chiral limit $m_u = m_d = m_s = 0$). This theory has the global symmetry group $G = \mathrm{SU}(3)_L \times \mathrm{SU}(3)_R \times \mathrm{U}(1)_V$, free of anomalies under the colour group. The $\mathrm{U}(1)_A$ factor is anomalous under the gauge group $\mathcal{G}_{\mathcal{DC}}$ and so it is explicitly broken. The global group has anomalies $\mathrm{SU}(3)^2_{L,R} \cdot \mathrm{U}(1)_V$ and $\mathrm{SU}(3)^3_{L,R}$. Assuming that QCD confines and that the global group is linearly realised one has that the anomalies should be balanced by massless composite fermions (baryons). However one can show (see [21] for a complete proof) that the anomaly matching condition cannot be satisfied by composite fermion in QCD with three flavours of massless quarks and it is possible to conclude that the spontaneous breaking of the global symmetry group is necessary.

Neutrino masses

We can apply anomaly matching considerations to the Standard Model. The theory has a global flavour symmetry group $G_F = \left(\mathrm{SU}(3) \times \mathrm{U}(1)\right)^5$, one factor for each irreducible representation: left-handed quarks Q_L and leptons L_L, right-handed quarks U_R, D_R and right-handed electrons E_R.

The global symmetry is broken explicitly by the Yukawa interaction with the Higgs field, so that the anomaly matching cannot be applied directly to the group G_F. However, there are two unbroken accidental $\mathrm{U}(1)$ symmetries, namely baryon number

$$B = \frac{1}{3} \quad \text{(quarks)}, \qquad B = 0 \quad \text{(leptons)}$$

and lepton number

$$L = 0 \quad \text{(quarks)} \qquad L = 1 \quad \text{(leptons)}$$

Both these symmetries are anomalous under the electroweak symmetry group, with anomaly

$$\partial_\mu j^\mu_B = \partial_\mu j^\mu_L = \frac{3}{32\pi^2} g_2^2\, \varepsilon_{\mu\nu\rho\sigma} W_i^{\mu\nu} W_i^{\rho\sigma}$$

The two anomalies cancel if we consider the linear combination $(B-L)$:

$$B - L = \frac{1}{3} \quad \text{(quarks)} \qquad B - L = -1 \quad \text{(leptons)}$$

Therefore the symmetry $\mathrm{U}(1)_{B-L}$ is a valid global symmetry free of gauge anomalies (while $\mathrm{U}(1)_{B+L}$ is anomalous and thus broken at the quantum level).

To apply 't Hooft anomaly matching, let us consider the global anomaly $\mathrm{U}(1)^3_{B-L}$. If we do not include right-handed neutrinos we obtain a non-zero anomaly:

$$\mathrm{Tr}\big[Y^3_{B-L}\big] = 3\big(2Y_Q^3 - Y_u^3 - Y_d^3\big) + \big(2Y_L^3 - Y_e^3\big) = 1 \neq 0$$

The existence of this anomaly, together with the fact that the symmetry is not spontaneously broken, implies the existence of massless fermions. In the Standard Model these correspond to the left-handed neutrinos, which in fact are very light states that have been believed to be massless for long time.

In recent years, the observation of neutrino oscillations have suggested that the neutrinos do have a mass different from zero. In order to have a consistent theory, the Standard Model must be extended, and there are two possibilities:

- the anomaly in the $(B-L)$ current is cancelled by new particles. This is the case, for instance, if right-handed neutrinos are introduced, giving mass to both left and right-handed neutrinos through the Higgs mechanism
- the global symmetry $\mathrm{U}(1)_{B-L}$ is explicitly broken. This can be obtained, for instance, taking into account non-renormalizable operators: at the level of dimension 5 there is a single operator consistent with Lorentz invariance and the Standard Model gauge symmetry:

  $$\mathcal{O}_5 = M_{\nu,ij}\big(\bar{L}^i H^c\big)\big(L^j H^c\big)^\dagger$$

  This operator induces an explicit breaking of the lepton number (and of $\mathrm{U}(1)_{B-L}$) and gives mass to the left-handed neutrinos, once the Higgs field acquires its vacuum expectation value.

3.3 Models chiral under $\mathrm{SU}(N)_{\mathrm{DC}}$

The dynamics of a chiral dark sector, with fermions transforming in a complex reducible representation of $\mathrm{SU}(N)_{\mathrm{DC}}$, is expected to be much different from the one of the models based on real representations.

It has been suggested that a composite Lorentz scalar operator (a fermion bilinear) may acquire a vacuum expectation value, inducing the dynamical breaking of the gauge group and possibly also of the global (flavour) symmetry group of the original lagrangian. Indeed, in a chiral model it is not possible to write a mass term for the fermions, and this is equivalent to say that every Lorentz scalar bilinear in the fermions cannot be a singlet of the dark colour gauge group.

Once the original gauge group $\mathrm{SU}(N)_{\mathrm{DC}}$ is broken to a subgroup $\mathrm{SU}(N)'_{\mathrm{DC}}$, one has to decompose the representations of the original fermions in representations of the unbroken gauge group. If the model is still chiral the process can repeat, generating a cascade of gauge symmetry breakings with several energy scales. Since the running of the gauge coupling is logarithmic, the separation of scales can be exponential: therefore it could be possible to generate in this way a hierarchy of scales in a natural way. The chain of symmetry breaking ceases when we are left with a real model with respect to the unbroken dark gauge group, which then experience a confining dynamics; otherwise the dynamics can lead to a complete breaking of the original gauge group. This scenario has been proposed by Dimopoulos, Raby and Susskind in [38] and is referred to as *tumbling*.

In order to predict what is the pattern of breaking of the dark color group one needs a criterion for which bilinear Lorentz scalar acquires a vacuum expectation value, *i.e.* what is the form of the condensate. The maximally attractive channel (MAC) criterion [38] has been proposed and can be phrased as follows: the bilinear scalar operator that acquires a non-zero vacuum expectation values is the one corresponding to the most attractive channel in the single gluon exchange interaction among dark quarks. The interaction between two dark quarks transforming as irreducible representations r_1 and r_2 depends on the composite representation $r' \subset r_1 \times r_2$. For each channel, the single gluon exchange approximation gives a potential

$$V(r) = \frac{g^2}{2r}\left(C_2^{(r')} - C_2^{(r_1)} - C_2^{(r_2)}\right)$$

The most attractive channel corresponds to the one for which the factor $\left(C_2^{(r')} - C_2^{(r_1)} - C_2^{(r_2)}\right)$ is negative and large.

The pattern of condensation obtained through the MAC criterion agrees with that predicted independently through instantons methods [39].

An alternative to tumbling has been suggested by Peskin [39]: all the condensates could form at the same energy scale, with a unique breaking of the gauge group. If this is the case, what happens is that each irreducible representation forms a condensate in the respective maximally attractive channel (*i.e.* all the dark quark form condensates and all the condensates form at the same scale).

We stress that the previous scenarios are speculative, being based on the assumption that a Lorentz scalar condensate acquires a vacuum expectation values; no rigorous argument or lattice simulation confirming clearly these conclusion is available, due to difficulties in putting on the lattice chiral gauge theories. Some authors have argued [62, 63] that a Lorentz vector condensate $\langle\psi_a\sigma_\mu\psi_a^*\rangle$ (which can always be a gauge singlet) might form instead of the scalar condensate; in this case the gauge group could remain unbroken, while it is unclear if Lorentz invariance would be spontaneously broken or not [63].

3.3.1 Anomaly cancellation and asymptotic freedom constraints

Let us discuss what are the general constraint that models with a chiral non-Abelian dark sector must satisfy for consistency.

We consider here models with $\mathrm{SU}(N)_{\mathrm{DC}}$ gauge group and left-handed Weyl fermions transforming as complex irreducible representations with up to two indices, *i.e.* fundamental, symmetric and antisymmetric (and their complex conjugates). For $N_{\mathrm{DC}} = 3, 4$ there are some special cases: for $N_{\mathrm{DC}} = 3$ the antisymmetric representation is equivalent to the anti-fundamental, while for $N_{\mathrm{DC}} = 4$ the antisymmetric representation is real and thus equivalent to its complex conjugate.

For simplicity, we assume that there are no subset of fermionic fields which transform in real (reducible) representations of $\mathrm{SU}(N)_{\mathrm{DC}}$, *i.e.* if there is a field transforming as the representation r, there are no fields transforming as $\bar{r}$.

We denote by n_F the number of fundamental representations minus the number of anti-fundamental representations, and similarly for n_S and n_A.

Asymptotic freedom for $\mathrm{SU}(N)_{\mathrm{DC}}$ (for a discussion on the running of the coupling constant and the condition of asymptotic freedom see section 4.1 in the next chapter) requires

$$|n_F|+(N_{\mathrm{DC}}-2)|n_A|+(N_{\mathrm{DC}}+2)|n_S|< 11N_{\mathrm{DC}} \tag{3.5}$$

Since we are dealing with a chiral model the conditions of gauge anomaly cancellation give non-trivial constraints. In order to make the notation less cumbersome we discuss first the case of dark quarks charged under $\mathrm{U}(1)_Y \supset \mathcal{G_{SM}}$ only and leave the general case for later.

The condition involves the group theory factor $T^{(r)}$ (Dynkin index) defined as:

$$\mathrm{Tr}\left[T_a^{(r)} T_b^{(r)}\right] = T^{(r)} \delta_{ab}$$

where $T_a^{(r)}$ are the generators of the irreducible representation r. Moreover, we denote by $\dim(r)$ the dimension of the representation r. The anomaly coefficient is defined by equation 3.4. Here we use the convention that $A(r_i)$ is definite positive, since we are taking n_i already with a minus sign for conjugate representations.

Gauge anomaly cancellation than gives the following constraints:

$$\mathrm{SU}(N)^3_{\mathrm{DC}} \qquad \rightarrow n_F A(F) + n_A A(A) + n_S A(S) = 0 \tag{3.6a}$$

$$\mathrm{U}(1)_Y\,\mathrm{SU}(N)^2_{\mathrm{DC}} \rightarrow \sum_{i=1}^{|n_F|} q_i^F T(F) + \sum_{i=1}^{|n_A|} q_i^A T(A) + \sum_{i=1}^{|n_S|} q_i^S T(S) = 0 \tag{3.6b}$$

$$\mathrm{U}(1)_Y \mathrm{grav}^2 \qquad \rightarrow \sum_{i=1}^{|n_F|} q_i^F \mathrm{dim}(F) + \sum_{i=1}^{|n_A|} q_i^A \mathrm{dim}(A) + \sum_{i=1}^{|n_S|} q_i^S \mathrm{dim}(S) = 0 \tag{3.6c}$$

$$\mathrm{U}(1)^3_Y \qquad \rightarrow \sum_{i=1}^{|n_F|} (q_i^F)^3 \mathrm{dim}(F) + \sum_{i=1}^{|n_A|} (q_i^A)^3 \mathrm{dim}(A) + \sum_{i=1}^{|n_S|} (q_i^S)^3 \mathrm{dim}(S) = 0 \tag{3.6d}$$

Defining[6]

$$q_F = \sum_{i=1}^{|n_F|} q_i^F \qquad q_A = \sum_{i=1}^{|n_A|} q_i^A \qquad q_S = \sum_{i=1}^{|n_S|} q_i^S$$
$$p_F = \sum_{i=1}^{|n_F|} (q_i^F)^3 \qquad p_A = \sum_{i=1}^{|n_A|} (q_i^A)^3 \qquad p_S = \sum_{i=1}^{|n_S|} (q_i^S)^3$$

and using the group theory factors listed in table 3.2, equations (3.6) become

$$n_F + n_A(N_{\mathrm{DC}} - 4) + n_S(N_{\mathrm{DC}} + 4) = 0 \tag{3.7a}$$
$$q_F + q_A(N_{\mathrm{DC}} - 2) + q_S(N_{\mathrm{DC}} + 2) = 0 \tag{3.7b}$$
$$q_F + q_A\frac{(N_{\mathrm{DC}} - 1)}{2} + q_S\frac{(N_{\mathrm{DC}} + 1)}{2} = 0 \tag{3.7c}$$
$$p_F + p_A\frac{(N_{\mathrm{DC}} - 1)}{2} + p_S\frac{(N_{\mathrm{DC}} + 1)}{2} = 0 \tag{3.7d}$$

Using equations (3.7b) and (3.7c), the solutions for q_i can be expressed as a one parameter family of solutions:

$$\left(q_F, \;\; q_A = -\frac{N_{\mathrm{DC}} + 3}{2N_{\mathrm{DC}}} q_F, \;\; q_S = \frac{N_{\mathrm{DC}} - 3}{2N_{\mathrm{DC}}} q_F \right)$$

In particular, for $N_{\mathrm{DC}} \neq 3$, if one of the q_i is zero then also the other two must be zero. Moreover, $(q_F = p_F = 0, q_A = p_A = 0, q_S = p_S = 0)$ is always a solution for every N_{DC}.

Equations (3.5) and (3.7a), together, give a constrain that can be graphically represented as a polygon in a two dimensional space (for instance n_F vs n_S, with n_A given as a function of the other two by eq. (3.7a))). Considering dark quarks charged under $\mathrm{SU}(2)_{\mathrm{EW}}$ and $\mathrm{SU}(3)_{\mathrm{c}}$ gives the following further constraints:

- it is necessary to have an even number of electroweak doublets due to the $\mathrm{SU}(2)_{\mathrm{EW}}$ global anomaly discussed in section 3.2;
- the anomaly $\mathrm{SU}(3)^3_{\mathrm{c}}$ must be zero, giving a constraint analogous to equation 3.6a;

[6] We define also: $q_i := 0$ if $|n_i| = 0$ and similarly for p_i.

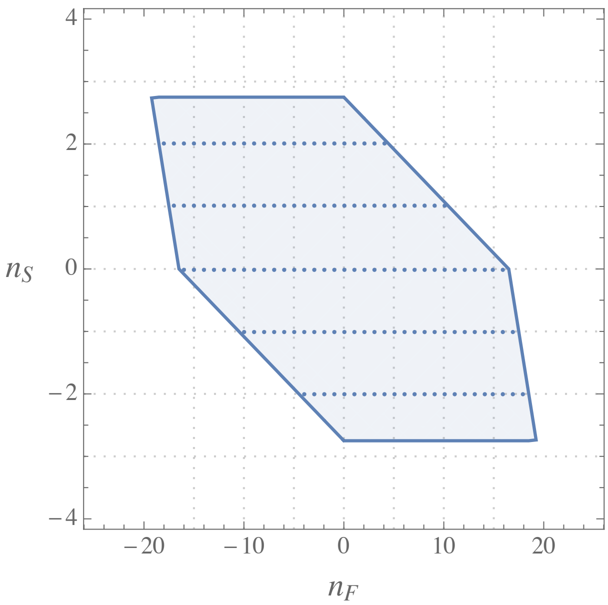


Figure 3.1: Combined bounds on n_F and n_S given by asymptotic freedom and anomaly cancellation, for $N_{\rm DC} = 3$. Anomaly cancellation fixes the number of antisymmetric representations to be $n_A = -(n_F + n_S(N_{\rm DC} + 4))/(N_{\rm DC} - 4)$.

Table 3.2: Group theory factors for irreducible representations with up to two indices of $\mathrm{SU}(N)$.

Irrep	Dimension dim(R)	Dynkin index T(R)	Quadratic Casimir C_2(R)	Anomaly Coefficient A(R)
Singlet	1	0	0	0
Fundamental	N	$\frac{1}{2}$	$\frac{N^2-1}{2N}$	1
Adjoint	N^2-1	N	N	0
Antisymmetric	$\frac{N(N-1)}{2}$	$\frac{N-2}{2}$	$\frac{(N-2)(N+1)}{N}$	$N-4$
Symmetric	$\frac{N(N+1)}{2}$	$\frac{N+2}{2}$	$\frac{(N-1)(N+2)}{N}$	$N+4$

- the mixed anomalies $\mathrm{U}(1)_Y\,\mathrm{SU}(2)^2_{\rm EW}$ and $\mathrm{U}(1)_Y\,\mathrm{SU}(3)^2_c$, must vanish, giving constraints analogous to equation 3.6b.

3.3.2 $\mathcal{G}_{\mathcal{SM}}$ chiralness

In this section we deal with the following question:

- is it possible to have a strongly interacting dark sector with field content *chiral* under $\mathcal{G}_{\mathcal{SM}}$ such that the dynamics does not break $\mathcal{G}_{\mathcal{SM}}$?

We have briefly discussed in section 3.1 that the pattern of spontaneous flavour symmetry breaking for models with real representations under $\mathcal{G}_{\mathcal{DC}}$ is such that the dynamics always breaks $\mathcal{G}_{\mathcal{SM}}$ if the field content is chiral under $\mathcal{G}_{\mathcal{SM}}$. For this reason in the literature on strongly interacting dark sectors, the fermions are assumed to be real under $\mathcal{G_{SM}}$ in such a way that there can be an orientation of the condensate that preserves $\mathcal{G_{SM}}$.

We want to understand if this condition is still necessary in the case of models chiral under $\mathcal{G_{DC}}$. The previous line of reasoning does not apply to this case, since now the condensate cannot be a singlet of $\mathcal{G_{DC}}$, therefore could be a singlet of $\mathcal{G_{SM}}$, depending on the quantum numbers.

We consider models completely chiral under $\mathrm{SU}(N)_{\mathrm{DC}}$, with dark quarks transforming as two indices complex representations of $\mathrm{SU}(N)_{\mathrm{DC}}$ for a generic N_{DC}, as described in the previous section.

To understand the dynamics of these models, we need to understand what is the pattern of symmetry breaking. Differently from the case of models with real representations, for which robust theoretical and numerical results are available (as discussed in section 3.1), this class of models is much less constrained on the theoretical ground and no observational or numerical clues are available. We stress that, in principle, understanding what is the pattern of symmetry breaking of the model is a well defined question with a unique solution determined by the dynamics; however, since no robust result is available, we shall make the following assumptions:

- the condensate occurs in the maximally attractive channel;
- the condensate does not mixes colour and flavour indices;
- the condensate preserves the maximal unbroken subgroup;

These assumptions are inspired by the pattern of symmetry breaking observed in models with real representations; we consider them as reasonable working hypothesis and use them in some clarifying examples. In order to advocate the viability of a concrete model, a robust analysis on the pattern of symmetry breaking in these models would be required.

Resting on this assumption we have tried to construct examples of models chiral under both $\mathcal{G_{DC}}$ and $\mathcal{G_{SM}}$, for which the dark sector dynamics leaves unbroken the Standard Model gauge group. Although we have succeeded in finding examples of models in which at the first step of the tumbling the condensates can preserve $\mathcal{G_{SM}}$, in our examples the further steps of the dynamics break $\mathcal{G_{SM}}$.

We have not a conclusive argument proving if this is possible or not; we present here the examples, sketching the dynamics of the first step of the tumbling.

Models with dark fermions charged under $\mathrm{U}(1)_Y$

Let us consider the following model:

$$\text{Gauge group}: \quad \mathrm{SU}(3)_{\mathrm{DC}} \times \mathrm{U}(1)_{\mathrm{Y}}$$
$$\text{Fields}: \quad (6)_0 + (\bar{3})_0 + (\bar{3})_0 + (\bar{3})_{-9} + (\bar{3})_{-5} + (\bar{3})_{-1} + (\bar{3})_7 + (\bar{3})_8$$

The hypercharges are chosen in order to have the minimal model which is chiral and anomaly free under $\mathrm{U}(1)_{\mathrm{Y}}$, and with integer charges. Indeed $q_S = p_S = 0$ is trivially verified and $q_A = p_A = 0$, as it can be readily checked (*i.e.* for each representation family, the sum of the hypercharges and the sum of the cubes of the hypercharge are both identically zero).

The model is also anomaly free, asymptotically free and chiral under $\mathrm{SU}(3)_{\mathrm{DC}}$. Indeed equations (3.6) and (3.7a) for $n_S = 1$, $n_F = -7$ and $N = 3$ give

$$\begin{aligned}
&11N > |n_F| + (N_{\mathrm{DC}} - 2)|n_A| + (N_{\mathrm{DC}} + 2)|n_S| \implies 11 \cdot 3 = 33 > 12 = 7 + 5 \cdot 1 \\
&n_F + n_A(N_{\mathrm{DC}} - 4) + n_S(N_{\mathrm{DC}} + 4) = 0 \implies -7 + 1 \cdot 7 = 0
\end{aligned}$$

The flavour group is given by $\mathrm{U}(1) \times \mathrm{SU}(7)$.

We assume that chiral symmetry breaking occurs, and that the condensate is given by the most attractive channel: $6 \times \bar{3} \to 3$.

Assuming, moreover, that the condensate does not mix flavour and colour indices (there is no colour-flavour locking), and that the maximal flavour subgroup is preserved (as for QCD), one has: $\mathrm{U}(1) \times \mathrm{SU}(7) \to \mathrm{U}(1) \times \mathrm{SU}(6)$. Just one linear combination of the dark quarks transforming in the anti-fundamental representation takes part in the condensate. It is then possible for the vacuum to choose an orientation such that the condensate has zero charge and $U(1)_Y$ is not spontaneously broken. This orientation is expected to be dynamically preferred through vacuum alignment as discussed in [32, 33].

The gauge group is broken dynamically by the condensate to a residual $\mathrm{SU}(2)_{DC}$. Decomposing the representations of the original gauge group in representations of the unbroken one, we are left with a model with real representations under $\mathcal{G}_{\mathcal{DC}}$ (indeed $\mathrm{SU}(2)$ has only real representation) and chiral under $\mathrm{U}(1)$, so that the dynamics break $\mathcal{G}_{\mathcal{SM}}$ at the second step of the tumbling.

The choice of the hypercharges is not unique and it arises in a non trivial way. Due to hypercharge quantisation one has to find integer solutions to a system of two homogeneous polynomial equations in order to have anomaly cancellation for the $\mathrm{U}(1)$:

$$\sum_i q_i^A = 0$$
$$\sum_i (q_i^A)^3 = 0$$

with the additional condition that there is at least an hypercharge $q_1^A \neq 0$ such that $q_i^A \neq -q_1^A$ for each i. Problem of this kind are notoriously difficult to solve. We have found some solutions numerically, that give alternative models with a dynamics similar to that of the one we just analysed:

Gauge group : $\mathrm{SU}(3)_{\mathrm{DC}} \times \mathrm{U}(1)_{\mathrm{Y}}$

Fields : $(6)_0 + (\bar{3})_0 + (\bar{3})_{-4} + (\bar{3})_{-4} + (\bar{3})_1 + (\bar{3})_1 + (\bar{3})_1 + (\bar{3})_5$

Gauge group : $\mathrm{SU}(3)_{\mathrm{DC}} \times \mathrm{U}(1)_{\mathrm{Y}}$

Fields : $(6)_0 + (\bar{3})_0 + (\bar{3})_{-6} + (\bar{3})_{-3} + (\bar{3})_2 + (\bar{3})_1 + (\bar{3})_5 + (\bar{3})_5$

Models with dark fermions charged under $\mathrm{SU}(2)_{\mathrm{EW}} \times \mathrm{U}(1)_Y$

The same mechanism is possible with a more generic choice of quantum numbers, if the chirality under $\mathcal{G}_{\mathcal{SM}}$ is obtained through a mixed use of $\mathrm{SU}(2)_{EW}$ and $\mathrm{U}(1)_Y$.

Let us consider the following model:

$$
\begin{aligned}
\text{Gauge group :} \quad & SU(3)_{DC} \times SU(2)_{EW} \times U(1)_Y \\
\text{Fields :} \quad & (6;2)_0 + (\bar{3};2)_0 + (\bar{3};2)_{y_1} + (\bar{3};2)_{y_2} + (\bar{3};2)_{y_3} \\
& + 2 \times (\bar{3};1)_{-y_1} + 2 \times (\bar{3};1)_{-y_2} + 2 \times (\bar{3};1)_{-y_3}
\end{aligned}
$$

We note that there is an even number of doublets $(6 + 3 \cdot 4 = 18)$ and therefore there is no global anomaly for the $\mathrm{SU}(2)_{\mathrm{EW}}$ gauge group. A further constraint is given by the requirement of mixed anomaly cancellation: $\mathrm{SU}(2)^2_{\mathrm{EW}} \cdot \mathrm{U}(1)_{\mathrm{Y}}$ implies the condition $y_1 + y_2 + y_3 = 0$. If the constraint is satisfied in a non-trivial way, *i.e.* the three hypercharges are taken all different from zero, then the model is chiral by construction. Moreover, it is asymptotically free as it can be checked using equation (3.5), anomaly free and chiral under $\mathrm{SU}(3)_{\mathrm{DC}}$.

The pattern of condensation and the dynamics are the same as the ones discussed of the previous example.

3.4 Flavour anomalies

We now consider in detail the global accidental symmetries of the dark colour dynamics and show that for chiral models with gauge group $\mathrm{SU}(N)_{\mathrm{DC}}$, in absence of fundamental scalar fields and if there are no further sources of breaking of the global symmetries, there are always asymptotic massless states.

Assuming that the dark colour dominates the dynamics at high energies, the flavour symmetry for a generic choice of quantum numbers (R_i, r_i) under $\mathrm{SU}(N)_{\mathrm{DC}} \times \mathcal{G}_{\mathcal{SM}}$ is

$$G_F = \bigotimes_{i=1}^{n} \mathrm{SU}(N_i) \bigotimes_{i=1}^{n-1} \mathrm{U}_i(1) \tag{3.8}$$

where n is the number of different dark color representations and N_j is the number of fermions that transform under the R_i. The $\mathrm{SU}(N)$'s mix fermions belonging to the same dark color representation. The $\mathrm{U}(1)$'s rotate each dark color representation with a phase. In principle there would be n $\mathrm{U}(1)$ factors; however there is always a linear combination that is anomalous under $\mathcal{G}_{\mathcal{DC}}$ and thus explicitly broken at the quantum level. The remaining $(n-1)$ have associated charges chosen in such a way to make them not anomalous under $\mathcal{G}_{\mathcal{DC}}$.

The flavour group has in general self anomalies G_F^3 and gravitational anomalies $G_F \cdot \mathrm{grav}^2$. For the 't Hooft anomaly matching, described in section 3.2, the anomalies in the UV must be reproduced in the IR; this is possible only if the exact physical spectrum includes massless fermions or if the symmetry is spontaneously broken (in which case there will be massless Goldstone bosons). The massless fermions can be either massless composite fermions (baryon-like), or elementary unconfined fermions if the gauge group is broken dynamically and the fermions of interest are singlet under the unbroken confining gauge group.

3.4.1 Flavour anomalies in $\mathrm{SU}(N)_{\mathrm{DC}}$ chiral gauge theories

We want to prove that for chiral models with an $\mathrm{SU}(N)_{\mathrm{DC}}$ gauge group and fermions charged under $\mathcal{G}_{\mathcal{SM}}$, there is a flavour anomaly G_F^3 or $G_F \cdot \mathrm{grav}^2$.

As said before, the fermions in the same dark colour representation have an $\mathrm{SU}(N_i)$ symmetry, where N_i is the multiplicity of the representation r_i. They transform as the fundamental representation of this global group. Moreover, there is a global $\mathrm{U}(1)$ factor for each distinct dark colour irreducible representation. Under this transformation the dark quarks transforming under the same irreducible representation share the same charge. One linear combination of the $\mathrm{U}(1)$ factors is explicitly broken by the anomaly under the dark colour gauge group.

A model with a unique anomaly-free complex irreducible representation of $\mathrm{SU}(N)_{\mathrm{DC}}$, would have no $\mathrm{U}(1)$ unbroken global symmetry. However for $\mathrm{SU}(N)$ gauge groups there are no representations with these properties with dimension smaller than $3 \cdot 10^5$ [64], therefore we consider models with two or more representation of small dimension.

We restrict our attention to chiral models with irreducible representations with up to two indices (*i.e.* fundamental, symmetric and antisymmetric representations and their complex conjugates).

The conditions that should be satisfied in order to have a consistent theory with an anomaly-free flavour group are the following:

1. $\mathrm{SU}(N)^3_{\mathrm{DC}}$ (gauge anomaly cancellation $n_F A(F) + n_A A(A) + n_S A(S) = 0$)
2. $U(1) \cdot \mathrm{SU}(N)^2_{\mathrm{DC}}$
3. $\mathrm{U}(1)^3_f$
4. $\mathrm{U}(1)_f \cdot \mathrm{grav}^2$
5. $\mathrm{SU}(N_f)^3$
6. $\mathrm{U}(1)_{f_1} \cdot \mathrm{U}(1)^2_{f_2}$
7. $\mathrm{U}(1)_{f_1} \cdot \mathrm{SU}(N_{f_2})^2$

In the scenario in which there are only two different representations, the conditions 2) - 4) become:

$$q_1|n_1|T(1) + q_2|n_2|T(2) = 0$$
$$q_1^3|n_1|\mathrm{dim}(1) + q_2^3|n_2|\mathrm{dim}(2) = 0$$
$$q_1|n_1|\mathrm{dim}(1) + q_2|n_2|\mathrm{dim}(2) = 0$$

The equations are homogeneous in the charges (they are scale invariant) and so we can choose an arbitrary normalization without loss of generality: $q_1 = 1$.

Subtracting the second and the third equation we obtain

$$q_2(q_2-1)(q_2+1)=0$$
$$q_2=-\frac{|n_1|\dim(1)}{|n_2|\dim(2)}<0$$

So we obtain $q_2=-1$. Using again the previous equations with $q_1=-q_2=1$ we arrive at[7]

$$\frac{T(1)}{T(2)}=\frac{A(1)}{A(2)}=\frac{\dim(1)}{\dim(2)}$$

Using the values of the group theory coefficients reported in table 3.2, we check if this equality is satisfied for the three possible couples: (fundamental, symmetric); (fundamental, antisymmetric); (symmetric, antisymmetric). We find that there are no solutions to the system.

We consider now the case with three different kinds of representations. If $N_i \geq 3$ for a given i, then we can conclude immediately that there is an $\mathrm{SU}(N_i)^3$ anomaly. To complete the proof, we show that if $N_i<3$ for each i than one of the two abelian $\mathrm{U}(1)$ factors is anomalous.

Let us focus on the $\mathrm{U}(1)$ factors of the flavour group and consider the conditions they must satisfy in order to be free of anomalies.

We make use the first four condition and see if there are solutions with $N_i<3$.

$$A_F n_F + A_A n_A + A_S n_S = 0 \tag{3.11a}$$
$$q_F|n_F|T(F) + q_A|n_A|T(A) + q_S|n_S|T(S) = 0 \tag{3.11b}$$
$$q_F^3|n_F|\dim(F) + q_A^3|n_A|\dim(A) + q_S^3|n_S|\dim(S) = 0 \tag{3.11c}$$
$$q_F|n_F|\dim(F) + q_A|n_A|\dim(A) + q_S|n_S|\dim(S) = 0 \tag{3.11d}$$

As for the case of two irreducible representations, we choose the normalization $q_F=1$. Moreover, dividing by $|n_F|$ and defining $x=n_S/n_F$, $y=n_A/n_F$ the system 3.11 becomes:

$$A_F + xA_S + yA_A = 0$$
$$T(F) + q_S|x|T(S) + q_A|y|T(A) = 0$$
$$\dim(F) + q_S|x|\dim(S) + q_A|y|\dim(A) = 0$$
$$\dim(F) + q_S^3|x|\dim(S) + q_A^3|y|\dim(A) = 0$$

For fixed number of dark colours N_{DC}, this is a system of four equations in four unknowns. However, the solutions are complicated functions of the parameters and do not take into account that the variable x and y can assume only rational values. Therefore we choose a different strategy: we concentrate only on the dangerous solutions with $|n_i|<3$.

The condition $|n_i|<3$ corresponds to the values $\pm 2, \pm 1, \pm 1/2$ for x and y. For each *dangerous* value of x and y, we solve the system as a function of q_S, q_A and N_{DC} (the system is over-constrained). If we found a solution with $N_{\mathrm{DC}}>2$ and integer, then we would have a model in

[7] We use the convention that $A(r_i)$ is definite positive, since we are taking n_i already with a minus sign for conjugate representations.

which the conditions of flavour and gauge anomaly cancellation 1) to 5) are all satisfied. In this case we should study the conditions 6) and 7) and see if they are satisfied or not. Numerically we find that there are no solutions with $N_{\rm DC} > 2$.

Since none of this solutions is acceptable, we conclude that the anomaly cancellation conditions cannot be completely satisfied and therefore that there are always 't Hooft anomalies in the flavour group of the models we are considering.

By 't Hooft anomaly matching there should be massless states in the physical spectrum, unless the flavour group is broken explicitly.

This argument is not valid if one has a single anomaly free chiral representation. For instance if one consider a single copy of the spinorial representation 16 of the gauge group $\mathrm{SO}(10)$, one has a chiral model with no unbroken global symmetry group, since the $\mathrm{U}(1)$ factor is anomalous under the gauge group. In this case one cannot use 't Hooft anomaly matching to conclude that there must be massless states in the infrared.

3.4.2 Mechanisms to give mass to the light states

We have shown that, if there exist at least two types of representations of $\mathrm{SU}(N)_{\rm DC}$ and in absence of elementary scalar fields, chiral models have non-vanishing flavour anomalies, that in turn implies the existence of massless states. We want now to understand which are the possible mechanisms that can give a non-vanishing mass to these light states.

In order to bypass the result of the flavour anomaly matching, an explicit breaking of the flavour symmetry group is required. We shall present an overview of some mechanism that can realise this and provide an estimate of the induced mass.

Yukawa coupling

A first possibility is to introduce a scalar field charged under the dark colour and with the right quantum numbers so that it is possible to write Yukawa coupling terms in the lagrangian. We stress that the scalar field has to be charged under $\mathcal{G}_{\mathcal{DC}}$ since in a chiral model it is not possible to write a fermion bilinear that is a singlet of $\mathcal{G}_{\mathcal{DC}}$.

In general these terms induce a breaking of the flavour group; if the scalar field acquires a vacuum expectation value different from zero this interaction induces a mass for the fermions of order yv.

This mechanism is the one that is realised in the Standard Model, where the Higgs field, charged under $\mathcal{G}_{\mathcal{SM}}$, gives mass to the fermions through the Yukawa interaction.

Gauging

When we consider the flavour group we are considering the global symmetry group of the lagrangian that describes the dark colour interaction, neglecting the Standard Model gauge interactions (assumed to give a small correction). This is analogous to what we do when we consider the QCD lagrangian and its global chiral symmetry group neglecting electroweak gauge interactions.

Switching on the Standard Model interactions induces, in general, an explicit breaking of the global flavour group G_F. The asymptotic states that transform under non-trivial representations of the gauge group receive loop corrections. For the fermion states we can estimate the contribution to their mass as

$$m \sim \frac{g^2}{16\pi^2}\Lambda_{\rm DC}$$

If the flavour group is spontaneously broken, the gauge interactions can generate a potential for the Goldstone bosons charged under $\mathcal{G}_{\mathcal{SM}}$. The contribution to their mass can be estimated as

$$m^2 \sim \frac{g^2}{16\pi^2}\Lambda_{\rm DC}^2$$

This is what happens in QCD for the charged pions $\pi^\pm$, whose mass gets a contribution from electromagnetic interactions.

Higher dimensional operators

The global flavour symmetry group is an accidental symmetry group of the renormalizable lagrangian. If we consider higher dimensional non-renormalizable operators we can induce a breaking of the global symmetry.

Axions and axion-like particles

Let us consider the Goldstone boson associated to the spontaneous breaking of the global symmetry group and suppose that the current associated to the corresponding generator has a mixed anomaly under the Standard Model gauge group.

This is for example the case for the pion: the current associate to the neutral pion π^0 has a mixed anomaly with electromagnetic interactions that induces the decay $\pi^0 \to \gamma\gamma$.

The anomaly induces a term in the effective action describing the pion

$$\mathcal{L} \supset \frac{\alpha_{em} N_{\rm c}}{12\pi F_\pi}\pi(x) F_{\mu\nu}\tilde{F^{\mu\nu}}$$

We want to understand if this term can induce a mass for the Goldstone boson.

Without including the term induced by the anomaly, the effective lagrangian describing the Goldstone boson has a shift symmetry which protects them from acquiring a mass. We want to understand if this shift symmetry is still a valid symmetry when we include the term induced by the anomaly. The point is that the shift induces a term which is analogous to the θ term. This term can be reabsorbed for $\mathrm{U}(1)_Y$ and for $\mathrm{SU}(2)_{\rm EW}$. Therefore, the shift symmetry is a valid symmetry that forbids the generation of a mass term (even by non-perturbative effects) for Goldstone bosons anomalous under $\mathrm{SU}(2)_{\rm EW} \times \mathrm{U}(1)_Y$.

The argument does not apply if there is an anomaly under the colour gauge group $\mathrm{SU}(3)_{\rm c}$. Indeed, in this case, the Goldstone boson plays the role of an axion and its mass can be computed through chiral lagrangian techniques [65, 66] to be

$$m_a^2 = \frac{m_u m_d}{(m_u + m_d)^2} \frac{m_\pi^2 f_\pi^2}{f_a^2}$$

where m_a is the axion mass and f_a is its decay constant where we expect the scaling law

$$f_a \sim \frac{\Lambda_{\rm DC}}{4\pi}$$

Chapter 4

Models with an infrared fixed point

We are interested in studying the dynamics and phenomenology of a model in which the dark sector has a non-trivial infrared fixed point and an explicit mass term of the fermions stops the flow towards the fixed point and gives a confining theory. The model is characterised by two scales, the dark quark mass M_Q and the confinement scale $\Lambda_{\rm DC}$, and exhibits a natural hierarchy of scales due to the non-trivial infrared dynamics. Furthermore, our choice for the fermionic representations has non-trivial phenomenological consequences that differentiate our model from others studied in the literature. Our scenario is a valid alternative to QCD-like models based on vector-like confinement and gives a different phenomenology, as we shall discuss in chapter 5.

In section 4.1 we briefly review the Renormalization Group flow and the definition of fixed point. We then discuss how a relevant term in the lagrangian, such as a mass term, can change the running of the coupling constant in a model with an infrared fixed point. We limit ourselves to the study of models in which the coupling at the fixed point is perturbative in order to have full control of the theory and perform reliable calculations, leaving the non-perturbative case for further studies.

In section 4.2 we show that an $\mathrm{SU}(N)_{\rm DC}$ gauge theory with 5 massless Weyl fermions in the adjoint representations has a perturbative infrared fixed point. We choose this model as a simple benchmark scenario and analyse its dynamics. We discuss what are the asymptotic states of the confining dynamics in the infrared, specialising to the case $N_{\rm DC} = 3$ in order to have quantitative clues from the existing lattice computations.

In the last section (4.3) we discuss what are the possible assignments of Standard Model quantum numbers for the dark sector fermions, compatible with the request that the SM gauge couplings do not have Landau poles at low energies. We then write the lagrangian for the two relevant models, discussing the masses of the different states, the accidental symmetries, and how higher dimensional operators break these symmetries.

4.1 Renormalization Group flow and infrared fixed points

In Quantum Field Theory the cancellation of ultraviolet divergences is achieved through the renormalization procedure [67]. The bare fields and parameters in the lagrangian are redefined through infinite renormalization constants, yielding to finite physical predictions. If the theory is renormalizable, a finite number of terms in the lagrangian is sufficient to absorb all the infinities, giving finite results for all the possible correlation functions needed in the calculation of physical observables.

The effects of the high momentum virtual particles running in the loops are encoded in the renormalization of the fields and parameters. As a result of the renormalization procedure, the parameters of the field theory turn out to be scale-dependent, with their evolution being described by differential equations known as *renormalization group* equations.

Formally, this can be seen as a consequence of the fact that in the renormalization procedure an arbitrary, unphysical, *renormalization scale* (usually denoted μ) must be introduced. Requiring that the physical observables are renormalization scale independent (setting the derivative of the observables with respect to the renormalization scale to zero) yields the renormalization group equations describing the *running* of the parameters as a function of the scale.

The evolution of the coupling constant as a function of the momentum, *i.e.* its *renormalization group flow*, is described by the β function

$$\beta(g) = \mu \frac{\mathrm{d}}{\mathrm{d}\mu} g(\mu) \tag{4.1}$$

The sign of the β function controls the qualitative behaviour of the running coupling:

- a positive sign of the β function describes a running coupling that increases at high energies (short distances) and decreases at small energies (long distances);
- a zero of the β function corresponds to a point at which the coupling constant does not flow and has a constant value, independent of the energy scale. This is called a *fixed point* of the renormalization group flow. The non-interacting free field theory ($g = 0$) has a trivial fixed point, β being identically zero;
- a negative sign of the β function describes a running coupling that increases at small energies (long distances) and decreases at high energies (short distances).

In theories in which $\beta(g)$ is non-negative, the infrared dynamics takes place in the neighbourhood of a fixed point, either trivial on non-trivial, while the short distance behaviour is non-perturbative. A prominent example is QED, a U(1) gauge theory with charged Dirac fermions: the infrared dynamics can be understood through perturbative computation while in the far ultraviolet there is a *Landau pole*, symptomatic of a non-perturbative dynamics taking place.

Conversely, theories in which the β function is negative are non-perturbative in the infrared regime and flow in the ultraviolet towards a fixed point. If the coupling constant flows to the trivial fixed point, then the theory is called *asymptotically free*. Theories in this class are completely under perturbative control in the short distance regime; a prominent example is given by QCD.

In the region in which the coupling constant is small enough, the β function can be computed in perturbation theory. Usually it is expressed as

$$\beta(g) = \mu \frac{\mathrm{d}}{\mathrm{d}\mu} g(\mu) = -b_0 \frac{g(\mu)^3}{16\pi^2} - b_1 \frac{g(\mu)^5}{(16\pi^2)^2} + \mathcal{O}\Big(g(\mu)^7\Big) \tag{4.2}$$

Let us now assume to be in the weak coupling regime, in which the three loop contribution can be neglected, and analyse the renormalization group flow of the model in different cases.

4.1.1 Asymptotically free models

If $b_0 > 0$ and $b_1 > 0$, the model is asymptotically free and flows towards the trivial fixed point in the ultraviolet. The qualitative behaviour of the running coupling constant in this case can be well approximated by neglecting the two loop contribution. The β function then becomes

$$\beta(g) = \mu \frac{\mathrm{d}}{\mathrm{d}\mu} g(\mu) = -b_0 \frac{g(\mu)^3}{16\pi^2} \tag{4.3}$$

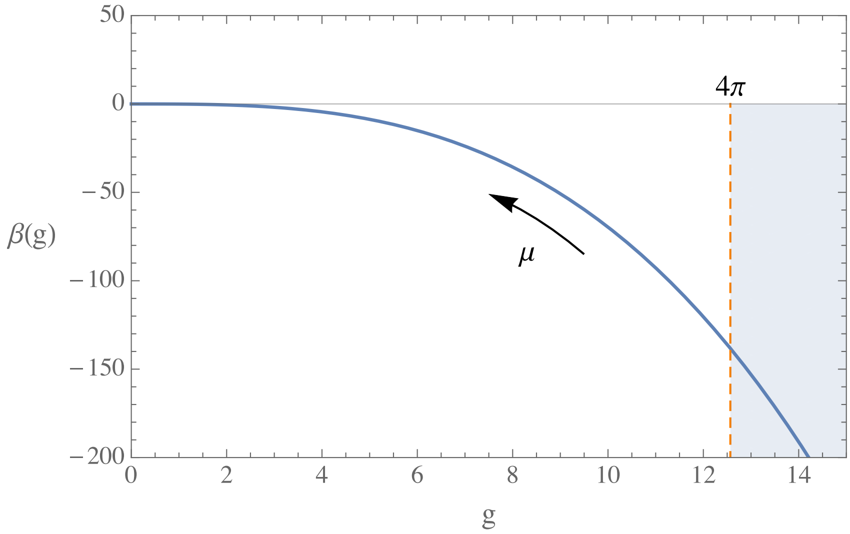
from which, integrating, we obtain

$$\frac{1}{g^2(\mu)} = \frac{b_0}{8\pi^2} \log\left(\frac{\mu}{\Lambda}\right) \quad \Longrightarrow \quad \frac{g^2(\mu)}{16\pi^2} = \frac{1}{2b_0 \log\left(\frac{\mu}{\Lambda}\right)} \tag{4.4}$$

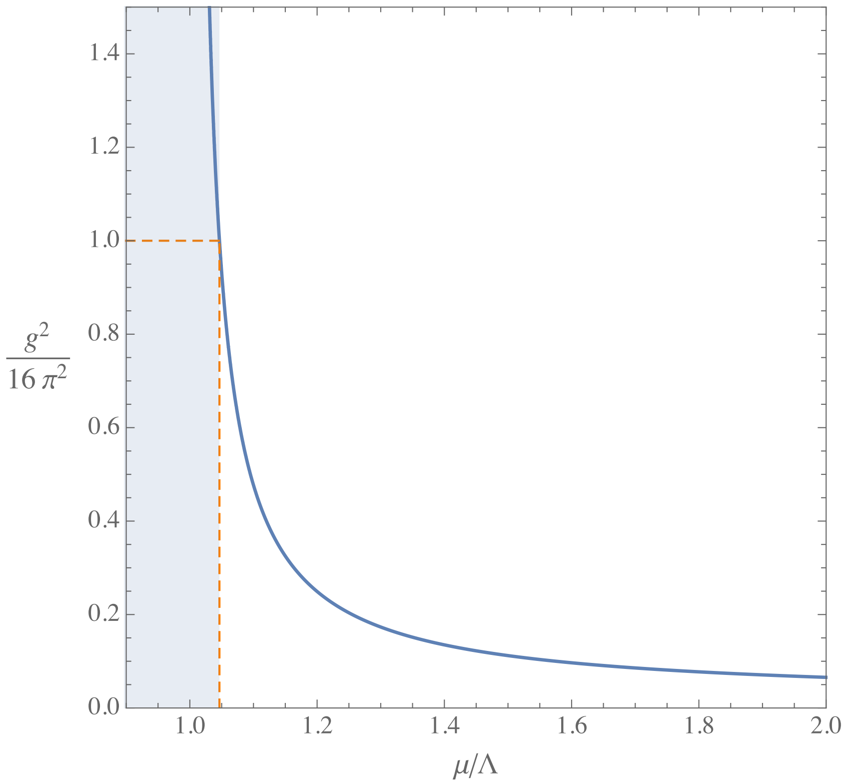
The theory can be defined either by assigning the value of the coupling constant g at a given scale μ_0 or by assigning the value of the energy scale Λ. The connection between these two point of views is called *dimensional transmutation* since it allows one to trade a coupling constant with a dimensionful parameter.

Even though the coupling seems to be divergent at the scale Λ, it should be noted that the solution 4.4 is valid only in the perturbative regime.

As an example, we plot in figure 4.1 the beta function and the evolution of the coupling constant for an SU(3) pure Yang Mills gauge theory, for which $b_0 = 11$.



(a) β function dependence on the coupling constant at one loop. The arrow denotes the orientation of the Renormalization Group flow with increasing energy.



(b) Running of the coupling constant at one loop. As it can be seen from the graph, the scale Λ roughly corresponds (up to order one factors) to the scale at which the dynamics becomes non-perturbative.

Figure 4.1: β function and running of the coupling constant in an asymptotically free model with no infrared fixed point (in the plot we use $b_0 = 11$, corresponding to the pure Yang Mills SU(3) gauge theory). The coupling constant decreases at high energy, approaching the trivial fixed point in the ultraviolet. The shaded area corresponds to the region in which the theory is non-perturbative and the perturbative calculation is no more valid.

4.1.2 Models with an infrared fixed point

Let us consider the case in which b_0 and b_1 have opposite sign. In the perturbative regime ($g \ll 4\pi$), assuming the three loops contribution to be negligible, the β function takes the form

$$\beta(g) = \mu \frac{\mathrm{d}}{\mathrm{d}\mu} g(\mu) = -b_0 \frac{g(\mu)^3}{16\pi^2} - b_1 \frac{g(\mu)^5}{(16\pi^2)^2} \tag{4.5}$$

A representative plot of the β function is shown in figure 4.2a.

The perturbative calculation would imply the existence of a zero of the β function, *i.e.* a fixed point $\beta(g_*) = 0$. In particular, since $b_0 > 0$ and $b_1 < 0$, the flow towards the fixed point is obtained for decreasing energies, both from above ($g(\mu) > g_*$) and from below ($g(\mu) < g_*$). Therefore, in this case the fixed point is said to be an *infrared* fixed point.

The value of the coupling at the fixed point would be given by

$$g_* = 4\pi \sqrt{-\frac{b_0}{b_1}} \tag{4.6}$$

However, we stress that this calculation can be trusted only if the coupling at the fixed point is perturbative $g_* \ll 4\pi$, that in turn implies $|b_0| \ll |b_1|$. Usually, in order for this to be the case, it is necessary to have a cancellation in the one-loop coefficient, so that the two-loop contribution is not a small correction in the neighbourhood of the fixed point. In the next section we shall show explicitly a model in which this is the case. For the moment let us just assume we are in this situation.

We point out that differently from the case discussed in the previous section, the condition $b_0 > 0$ does not assure that the model is asymptotically free. Indeed, this is true only in the region in which the β function is negative: the left branch of figure 4.1 corresponding to the condition $g(\mu) < g_*$. On the contrary, if $g(\mu) > g_*$ then the coupling grows with energy and the model has a Landau pole in the ultraviolet. In order to completely define a model it is necessary to specify the value of the coupling constant at a given scale and this assignment specifies in which branch we are.

In the neighbourhood of the fixed point the β function can be approximated as

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} g(\mu) = a(g - g_*) \tag{4.7}$$

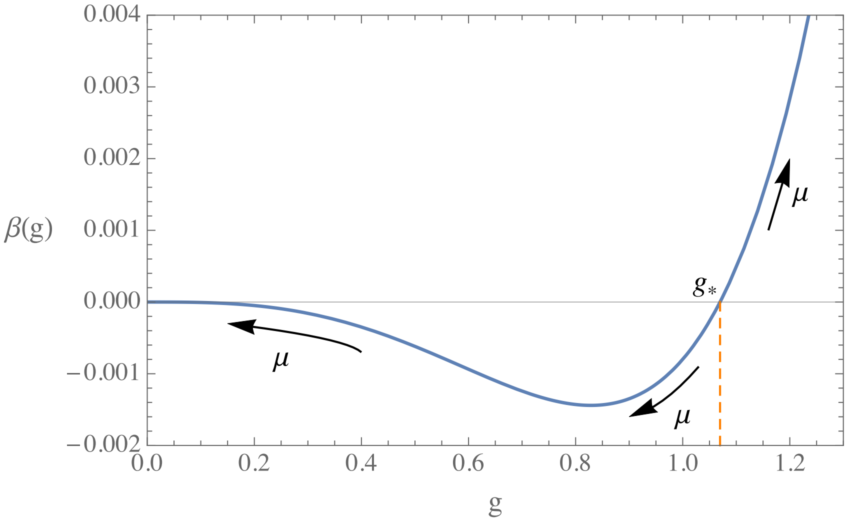
where

$$\beta(g_*) = 0, \qquad a = \frac{\mathrm{d}}{\mathrm{d}g}\beta(g_*) = -\frac{2b_0^2}{b_1}$$

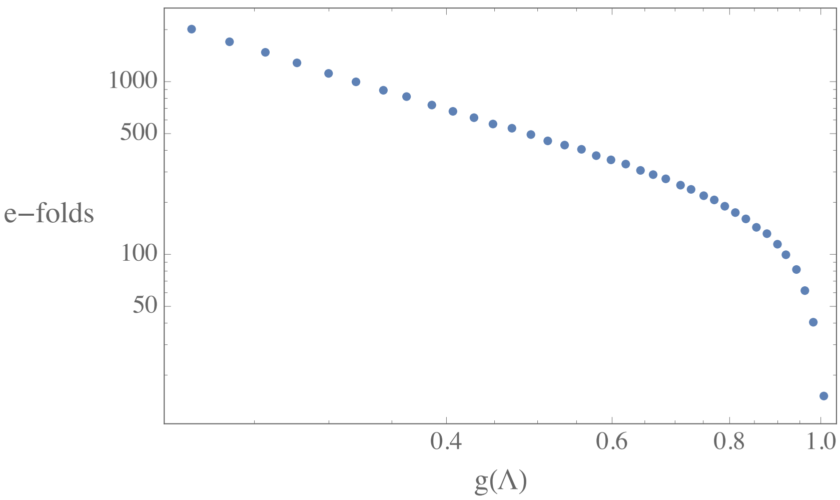
A model with an infrared fixed point has $a > 0$, while a model with an ultraviolet fixed point would have $a < 0$.

Equation 4.7 can be readily integrated, obtaining:

$$g(\mu) = g_* + \Big(g(\Lambda) - g_*\Big)\left(\frac{\mu}{\Lambda}\right)^a \tag{4.8}$$



(a) β dependence on the coupling constant at two loops. The coupling at the fixed point is denoted by g_* and it is perturbative. The arrows denote the orientation of the Renormalization Group flow with increasing energy. As it can be seen, there are two branches: the left one corresponds to an asymptotically free model with an infrared fixed point; the right one corresponds to a model with a Landau pole in the UV and a fixed point in the infrared.



(b) Number of e-folds necessary to be near the fixed point as a function of the initial value. Specifically, we plot $-\log(\mu/\Lambda)$ where $|g(\mu) - g_*| = 0.05$ and $g(\Lambda)$ is the initial condition, solving numerically the two loop equation. As can be seen from the graph, the running for this model is very slow; the reason is that in this model the β function in the asymptotically free branch is bounded by $|\beta| < 0.0015$, a very small value.

Figure 4.2: β function and running of the coupling constant in a model with an infrared fixed point and massless fermions. In the graph we use $b_0 = 1$ and $b_1 = -138$, corresponding to the model described in section 4.2 - an SU(3) gauge theory with 5 Weyl fermions in the adjoint.

Not asymptotically free branch

In the branch at the right of the fixed point in figure 4.2 the β function is positive and the model is no longer asymptotically free but has a Landau pole in the ultraviolet.

This is the case because in this region the two loops contribution to the β function, which is positive for a model with an infrared fixed point, starts to dominate the evolution.

In order to have a viable model we require the Landau pole to be above the Planck mass scale $M_{\rm Pl} = 1.2 \times 10^{19}\,\text{GeV}$. If we define our model by assigning the value of the coupling at a given scale, this request results in an upper bound on the value that the coupling can have. In order to have a quantitative bound, we do an order of magnitude estimate of the Landau pole scale by approximating the β function with the two loop contribution alone, which is the one that dominates in the region in which $\beta > 0$.

Then we have

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} g(\mu) = -b_1 \frac{g(\mu)^5}{(16\pi^2)^2} \tag{4.9}$$

and integrating we obtain:

$$\frac{1}{g^4(\mu)} = \frac{b_1}{2^{10}\pi^4} \log\left(\frac{\mu}{\Lambda_{\rm LP}}\right) \tag{4.10}$$

From this, requiring that $\Lambda_{\rm LP} > M_{\rm Pl}$, we obtain an upper bound on the coupling:

$$g(\mu) \leq \left(\frac{2^{10}\pi^4}{b_1 \log\left(\frac{\mu}{M_{\rm Pl}}\right)} \right)^{\frac{1}{4}} \tag{4.11}$$

By including the one loop contribution, which is negative, the estimate for the upper bound would increase, since the running would be slower. In our analysis, however, it will be sufficient to use the lower estimate, which gives a more restrictive bound, as we shall not work on the boundary of this region.

4.1.3 Conformal window

To be concrete, let us consider an $\mathrm{SU}(N)_{\rm DC}$ gauge theory with N_f Weyl fermions transforming as a representation r of the gauge group[1]. The β function coefficients are given by [68, 69]:

$$b_0 = \frac{11}{3} C_2^{(adj)} - \frac{2}{3} T^{(r)} N_f \tag{4.12}$$

$$b_1 = \frac{34}{3}\left(C_2^{(adj)}\right)^2 - 2T^{(r)} N_f \left[\frac{5}{3} C_2^{(adj)} + C_2^{(r)}\right] \tag{4.13}$$

where $C_2^{(r)}$ is the quadratic Casimir invariant of the representation (r) defined by

$$C_2^{(r)} \equiv \sum_a T_a^{(r)} T_a^{(r)}$$

[1] N_f is the multiplicity of the representation r, often referred to as the number of *flavours*

and $T^{(r)}$ is the Dynkin index of the representation defined as

$$T^{(r)}\delta_{ab} \equiv \mathrm{Tr}\left[T_a^{(r)}T_b^{(r)}\right]$$

Using the explicit value of the Casimir invariant for the adjoint representation $C_2^{(adj)} = N_{\rm DC}$, we obtain:

$$b_0 = \frac{11}{3}N_{\rm DC} - \frac{2}{3}T^{(r)}N_f \tag{4.14}$$

$$b_1 = \frac{34}{3}(N_{\rm DC})^2 - \frac{2}{3}T^{(r)}N_f\left[5N_{\rm DC} + 3C_2^{(r)}\right] \tag{4.15}$$

Varying the number of dark colours and the number of flavours, the β function coefficient can change sign. Let us consider the behaviour of the two coefficients as a function of the number of flavours N_f for a fixed number of colours $N_{\rm DC}$.

The one-loop coefficient b_0 is positive for $N_f = 0$ and, since $T^{(r)} > 0$, it decreases with N_f. At a certain point, b_0 becomes negative and asymptotic freedom is lost: we define $(N_f)_{af}$ to be the value (not necessarily integer) at which the zero is attained. One finds

$$(N_f)_{af} = \frac{11}{2}\frac{N_{\rm DC}}{T^{(r)}}$$

Similarly, the two loop coefficient b_1 is positive for $N_f = 0$ and decreases with N_f. We define $(N_f)_*$ as the value at which the zero occurs. We have

$$(N_f)_* = \frac{17(N_{\rm DC})^2}{T^{(r)}\left(5N_{\rm DC} + 3C_2^{(r)}\right)}$$

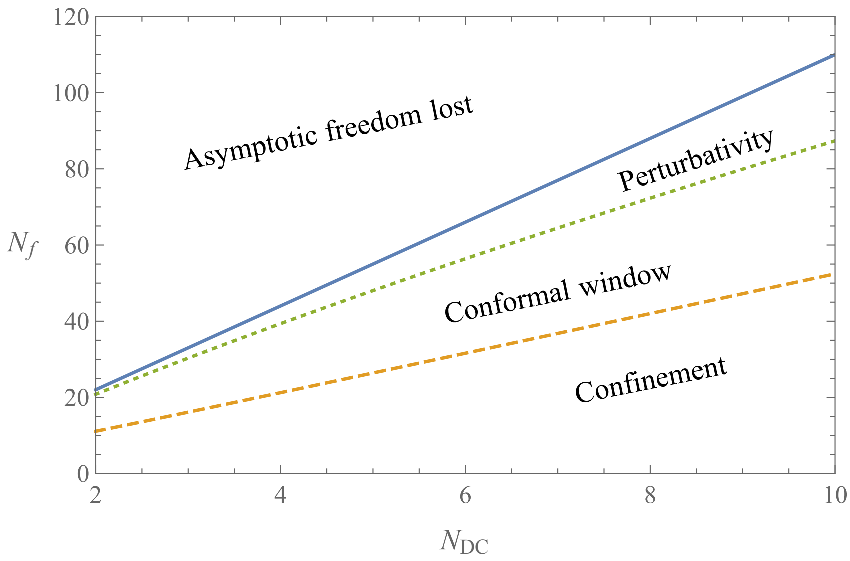
At fixed number of colours $N_{\rm DC}$, one finds $(N_f)_{af} > (N_f)_*$ as can be readily verified; therefore, there is always a region in which $b_0 > 0$ and $b_1 < 0$. As we discussed in the previous paragraph, the perturbative analysis implies the existence of a fixed point in the region in which the coupling g_* is perturbative. We have

$$\frac{g_*^2}{16\pi^2} = \frac{11(N_{\rm DC}) - 2TN_f}{2TN_f\left(5N_{\rm DC} + 3C_2^{(r)}\right) - 34N_{\rm DC}^2}$$

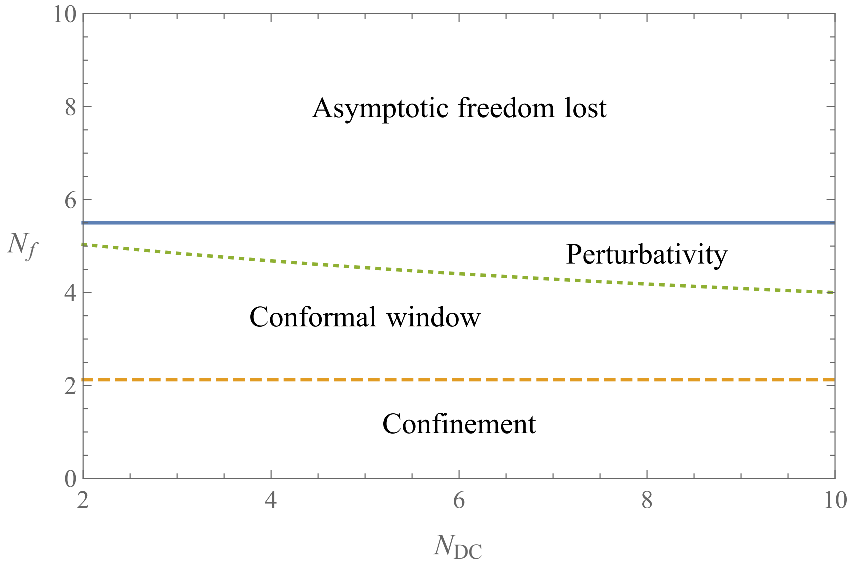
To be conservative, we consider as perturbative the region in which $g_*^2/16\pi^2 \lesssim 1/100$. The existence of a fixed point can extend beyond the perturbative regime; the region in which the theory has an infrared fixed point is referred to as the *conformal window*. Lattice simulations are the main tool available to analyse the behaviour of the theory in the non-perturbative regime[2], and many works have been devoted to finding the lower boundary of the conformal window (for a review see [36], while for a summary of lattice results see [51]).

The phase structure of non-Abelian gauge theories with vectorlike fermions has been studied

[2]With the exception of supersymmetric theories, for which exact non-perturbative results have been derived by analytical methods.



(a) Weyl fermions in the fundamental representation.



(b) Weyl fermions in the adjoint representation.

Figure 4.3: Conformal window as a function of the number of dark colours and the number of Weyl fermions for two different representations. The conformal window encompasses the region in which the coupling is perturbative; however, the boundary between the conformal window and the confinement region is non-perturbative and the graph is only representative.

first by Banks and Zaks in [70] and the fixed point is often called *Banks-Zaks fixed point.*

Representative plots are shown in figure 4.3 for Weyl fermions in the fundamental and the adjoint representations.

4.1.4 Mass term and breaking of the approximate conformal symmetry

We are interested now in describing the dynamics of the model in the vicinity of the fixed point and to see what happens if there is a deformation such as a mass term.

A model is said to have a conformal symmetry if the stress-energy tensor is traceless, *i.e.* $\Theta^\mu_\mu = 0$. In particular, it is possible to show that if this condition is satisfied, then the model is scale invariant [67]. A gauge theory with massless fermions and only marginal terms in the lagrangian is scale invariant at the classical level, since there is no mass scale. This invariance is however broken at the quantum level by the so called trace anomaly[3] [67]:

$$\Theta^\mu_\mu = \beta(g)\frac{\partial}{\partial g}\mathcal{L} \tag{4.16}$$

In the region in which the β function is negligible (*i.e.* near a fixed point) we have a model with an approximate conformal invariance.

Let us introduce now a deformation to this model by adding a relevant term in the lagrangian:

$$\mathcal{L} \supset c\Lambda^{4-\Delta}\mathcal{O}_\Delta$$

where Δ is the scaling dimension of $\mathcal{O}_\Delta$ and we require $\Delta < 4$ in order to have a relevant term. For example this could be the mass term of a fermion: $\mathcal{O}_\Delta = \psi\psi$. If the model is weakly interacting, for this operator we have[4]: $\Delta \sim 3$ and $c\Lambda^{4-\Delta} = M_\mathcal{Q}$. However, if the dynamics is strong and we are near the fixed point, the anomalous dimension can be large.

Let us assume that at a given energy Λ_{UV} the β function and the coefficient in front of the relevant operator in the lagrangian are both small $\beta \ll 1$, $c_{\text{UV}} \ll 1$, so that the model exhibits an approximate conformal symmetry.

Then, flowing at low energies towards the infrared fixed point, the coefficient of the relevant term grows like:

$$c(\mu) = c_{\text{UV}}\left(\frac{\Lambda_{\text{UV}}}{\mu}\right)^{4-\Delta}$$

At a scale Λ_{IR} the coefficient becomes of order 1 and the approximate conformal invariance is explicitly broken by this term. This happens at:

$$\Lambda_{\text{IR}} = (c_{\text{UV}})^{\frac{1}{4-\Delta}}\Lambda_{\text{UV}} \tag{4.17}$$

If the dynamics near the fixed point is perturbative, then the dimension of the operator is approximately the classical one $\Delta \simeq 3$ and equation 4.17 becomes

$$\Lambda_{\text{IR}} = c_{\text{UV}}\Lambda_{\text{UV}} = M_\mathcal{Q}$$

Below the scale $M_\mathcal{Q}$ the heavy fermions decouple and we are left with an effective gauge theory with an effective coupling [71]. At tree level, neglecting threshold effects, the matching between the effective coupling and the true coupling must be done at the energy $M_\mathcal{Q}$.

If, for instance, the fermions share all the same mass, then we are left with an effective theory which is a pure gauge theory that confines in the infrared. The running is described by

[3]That is to say: at the quantum level a scale arises through dimensional transmutation.

[4]This means that the anomalous dimension is a small correction with respect to the classical dimension of the operator.

equation 4.4, where $b_0 = \frac{11}{3} N_{\mathrm{DC}}$ for a pure gauge theory. Inverting this equation we obtain

$$\Lambda_{\mathrm{DC}} = M_{\mathcal{Q}} \exp\left(-\frac{8\pi^2}{b_0 g(M_{\mathcal{Q}})^2}\right) \tag{4.18}$$

If the fixed point is perturbative, or if the coupling at the scale $M_{\mathcal{Q}}$ is perturbative, a great separation of scales between $M_{\mathcal{Q}}$ and Λ_{DC} can be obtained in a natural way.

From the low energy point of view we have then a confining gauge theory with fermions heavier than the confinement scale and a natural hierarchy between the mass and the confinement scale.

4.2 Model with adjoint fermions

We consider here an $\mathrm{SU}(N)_{\mathrm{DC}}$ gauge theory with N_f Weyl fermions transforming as the adjoint representation of the gauge group.

For the adjoint representation, the group theory invariants $C_2^{(adj)}$ and $T^{(adj)}$ have both the value N_{DC}. Therefore, from equation 4.12 we obtain

$$\begin{aligned} b_0 &= \frac{11}{3} N_{\mathrm{DC}} - \frac{2}{3} N_{\mathrm{DC}} N_f \\ b_1 &= \frac{34}{3} N_{\mathrm{DC}}^2 - 2 N_{\mathrm{DC}} N_f \left[\frac{5}{3} N_{\mathrm{DC}} + N_{\mathrm{DC}}\right] = \frac{34}{3} N_{\mathrm{DC}}^2 - \frac{16}{3} N_{\mathrm{DC}}^2 N_f \end{aligned} \tag{4.19}$$

We stress that the one loop and two loops coefficients of the β function are renormalization-scheme independent.

In order to have an infrared fixed point it is necessary to have

$$\begin{aligned} b_0 > 0 &\Rightarrow N_f < \frac{11}{2} \\ b_1 < 0 &\Rightarrow N_f > \frac{17}{8} \end{aligned} \tag{4.20}$$

However the perturbative calculation is valid only if the value of the coupling at the fixed point is perturbative $(g_*^2/16\pi^2) \ll 1$.

The value of the coupling at the fixed point is

$$g_* = 4\pi\sqrt{-\frac{b_0}{b_1}} = 4\pi\sqrt{\frac{11 - 2N_f}{15 N_f N_{\mathrm{DC}} - 34 N_{\mathrm{DC}}}} \tag{4.21}$$

which is a decreasing function of the number of flavours N_f.

The conditions 4.19 are satisfied for $N_f = 3, 4, 5$, therefore the perturbative calculation would suggest an infrared fixed point for these models.

The case $N_f = 4$, which can be equivalently seen as a vector-like model with two flavours of adjoint Dirac fermions, has been studied through lattice simulations; the results indicate a non-perturbative dynamics: for $N_{\mathrm{DC}} = 2$ the model is in the conformal window [72], while for $N_{\mathrm{DC}} = 3$ there is confinement [73, 74]. This suggests that the models with $N_f = 3, 4$ have a non-perturbative dynamics. The model with $N_{\mathrm{DC}} = 2$ and $N_f = 4$ has been studied in the context of technicolor theories for the electroweak symmetry breaking, in particular in the works Minimal Walking Technicolor [75].

For $N_f = 5$, there is a cancellation between the two terms of the coefficient b_0, leading to the smallest value for the coupling g_*. We have:

$$
\begin{aligned}
b_0 &= \frac{1}{3} N_{\mathrm{DC}} \\
b_1 &= -\frac{46}{3} N_{\mathrm{DC}}^2 \\
g_* &= 4\pi \sqrt{\frac{1}{46 N_{\mathrm{DC}}}} \approx \frac{1.85}{\sqrt{N_{\mathrm{DC}}}}
\end{aligned}
$$

For $N_{\mathrm{DC}} = 3$, the value of the coupling at the fixed point is $g_* \approx 1.07$. The case $N_f = 5$ is more difficult to study on the lattice with the usual techniques since we are dealing with a gauge theory which is not vector-like. At the moment there are no available lattice simulations; since $(g_*^2/16\pi^2) \ll 1$, we assume the perturbative calculation to be valid and that there is an infrared fixed point.

The existence of a perturbative fixed point for this choice of representations is known in the literature. However phenomenological studies based on models with conformal dynamics have been conducted mainly in the context of technicolor theories describing the electroweak symmetry breaking (for a review see [36]). At our knowledge, no concrete phenomenological model based on the choice of representations with 5 adjoint Weyl fermions has been studied yet, in particular in the context of dark sectors.

We shall consider this model as a benchmark scenario for a dark sector with an infrared fixed point and study its phenomenology, that, as we shall see, is vastly different from the one realised in models realising vector-like confinement.

An other possible choice would be to study the phenomenology of a dark sector with an $\mathrm{SU}(N)_{\mathrm{DC}}$ gauge theory near the Banks-Zaks fixed point [70]. However, to be near the Banks-Zaks fixed point in the perturbative region, 16 Dirac fermions (*i.e.* 32 Weyl fermions) in the fundamental representation of the gauge group are needed. This is a huge multiplicity compared with the 5 Weyl fermions that are sufficient in the adjoint case. Furthermore, we find interesting to study the phenomenology of a model with fermions in the adjoint which is much different and has not received much attention in the literature on non-abelian dark sectors until now.

4.2.1 Dynamics of the model

From now on, we shall focus on the model with gauge group $\mathrm{SU}(N)_{\mathrm{DC}}$ and 5 Weyl fermions transforming as the adjoint. To be concrete, we specialise to the case $N_{\mathrm{DC}} = 3$. We shall refer to the gauge bosons of $\mathrm{SU}(3)_{\mathrm{DC}}$ as *dark gluons* and to the fermions charged under this group as *dark quarks*.

Using the results of the previous section, for this model we obtain $b_0 = 1, b_1 = -138$. The β function is shown in figure 4.2.

From equation 4.6, the coupling at the fixed point is $g_* = 1.07$.

The running in the asymptotically free branch is very slow. This is due to the fact that the β function in this branch is bounded by $|\beta| < 0.0015$, a very small value. It follows that, for example, to have an increase $\Delta g = 0.1$ in the coupling constant the running scale should decrease by 29 orders of magnitude.

In fact, such large excursions are never obtained. In practice, the coupling has an approximately constant value in the mass range of interest. We consider the Planck mass $M_{\rm Pl}$ as the ultraviolet cut-off of the model. In the infrared, the coupling flows towards the fixed point until the energy becomes smaller than the mass of the dark quarks. Let us assume for simplicity that the 5 adjoint fermions all share the same mass. Then, below this threshold the fermions decouple and the dynamics can be described by an effective pure gauge theory.

To define the model we specify the value of the coupling at the mass scale of the fermions.

In the asymptotically free branch we have $0 < g(M_{\mathcal{Q}}) \leq g_* = 1.07$. The confinement scale is given by equation 4.18. In this branch due the exponential suppression we obtain a large hierarchy between the mass and the confinement scale:

$$\Lambda_{\rm DC} \leq M_{\mathcal{Q}} \exp\left(-\frac{8\pi^2}{b_0|_{\rm YM}\, g_*^2}\right) = 2 \times 10^{-3} M_{\mathcal{Q}} \tag{4.22}$$

where $b_0|_{\rm YM}$ is the β function one-loop coefficient of the pure gauge infrared theory. For an initial condition $g(M_{\mathcal{Q}}) = 0.5$ we would have a confinement scale $\Lambda_{\rm DC} \sim 10^{-13} M_{\mathcal{Q}}$.

In the second branch $g(M_{\mathcal{Q}}) \geq g_*$. The upper bound 4.11 for a typical fermions mass scale of $M_{\mathcal{Q}} \sim \mathcal{O}(1-10)$ TeV becomes $g(M_{\mathcal{Q}}) \lesssim 2.1$. In this case there can be a smaller separation of scales among $\Lambda_{\rm DC}$ and $M_{\mathcal{Q}}$. The upper bound on the coupling constant translates into an upper bound on the confinement scale:

$$\Lambda_{\rm DC} \leq M_{\mathcal{Q}} \exp\left(-\frac{\left(b_1 \log\left(\frac{M_{\mathcal{Q}}}{M_{\rm Pl}}\right)\right)^{\frac{1}{2}}}{4\, b_0|_{\rm YM}}\right) = M_{\mathcal{Q}} \exp\left(-2.9\sqrt{\log\left(\frac{M_{\rm Pl}}{M_{\mathcal{Q}}}\right)}\right) \tag{4.23}$$

For $M_{\mathcal{Q}} \sim \mathcal{O}(1-10)$ TeV this gives $\Lambda_{\rm DC} \lesssim 0.2\, M_{\mathcal{Q}}$.

Bound states

We have a confining gauge theory with heavy dark quarks transforming as the adjoint representation.

The dark gluons form bound states known as glueballs with a mass of order $\Lambda_{\rm DC}$. The ligthest glueball, Φ, has quantum numbers $J^{PC} = 0^{++}$ and a mass $M_\Phi \approx 7\, \Lambda_{\rm DC}$ [76–78].

In the heavy quark regime $M_{\mathcal{Q}} \gg \Lambda_{\rm DC}$, we expect the mass of the bound states containing at least one dark quark to be roughly the sum of the masses of the constituent quarks, with a negligible contribution from the binding energy.

In the case of adjoint fermions the lightest state involving a quark is a bound state of one quark and one gluon. Indeed since they both transform as the adjoint we have the following decomposition

$$\mathbf{8} \otimes \mathbf{8} = \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{10} \oplus \overline{\mathbf{10}} \oplus \mathbf{27} \tag{4.24}$$

and we see that it is possible to have a dark colour singlet. The bound state, which we shall call $\mathcal{X}$, has a mass of order $M_{\mathcal{X}} \approx M_{\mathcal{Q}}$, spin $\frac{1}{2}$ and the same Standard Model quantum numbers as its constituent dark quark. Its existence is confirmed by lattice calculations [79, 80]. In the context of supersymmetric theories in which gluons come with gluinos (adjoint fermions), this bound state is known as glueballino. We shall call it *gluequark* to remark that we are dealing with a non-supersymmetric model.

As we shall discuss in the next section, the models we shall be considering feature an accidental $\mathbb{Z}_2$ symmetry[5] acting on the dark quark fields, referred to as *dark parity.* Summarising, the asymptotic states can then be classified according to this symmetry:

- dark parity even states are composite states with an even number of constituent dark quarks. The lightest state which in this group is the glueball Φ with quantum numbers $J^{PC} = 0^{++}$; lattice simulations suggest a mass $M_\Phi \approx 7\ \Lambda_{\rm DC}$.
- dark parity odd states have an odd number of dark quarks. The lightest state in this group is the gluequark $\mathcal{X}$ which in the heavy quark regime has a mass $M_{\mathcal{X}} \approx M_{\mathcal{Q}}$

Dark quarks can also form bound states with two or more constituent quarks, such as di-quarks (the analogue of mesonic states) and tri-quarks (the analogue of baryonic states for an SU(3) gauge theory). Nevertheless, these states are heavier and are expected to decay to glueballs and gluequarks , the lightest states with odd and even dark parity. Indeed, the multi-quark states are not protected by any further symmetry and the decay modes are kinematically allowed (in the regime $M_{\mathcal{Q}} \gg \Lambda_{\rm DC}$). This is a crucial difference with respect to the case of fermions transforming as the fundamental representation, where dark mesons and dark baryons are the relevant asymptotic states with constituent dark quarks.

4.3 Standard Model quantum numbers and spectrum

We are interested in studying the possibility that the dark sector interacts with the Standard Model fields not only through gravity but also through electroweak interactions. For our model we have then to understand what are the possible assignments of Standard Model quantum numbers for the new fields.

The introduction of new fermions charged under the Standard Model gauge group modifies the running of the couplings[6]. The main restriction arises from the requirement that the Standard Model gauge couplings do not have Landau poles below the Planck mass.

The new particles charged under the electroweak group modify the running at scales above their mass scale $M_{\mathcal{Q}}$. Let b^{SM} be the one loop β function coefficient predicted by the Standard

[5]This is an accidental global symmetry of the renormalizable lagrangian, but could be broken explicitly by higher dimensional operators.

[6]It has been proposed in reference [81] to use the energy dependence of the coupling constants α_1, α_2 to set limits on new particles with electroweak interactions in a model independent way. However, the current sensitivity is limited to new states of mass $M_{\mathcal{Q}} \leq 1\,\text{TeV}$ (see figure 17 in [81]). As we shall see, in our models the dark matter candidates have a mass greater than 1 TeV, avoiding the collider bounds on the running of the electroweak couplings; future experiments such as a 100 TeV proton-proton collider can be sensitive to the mass range of interest.

Model and let Δb be the contribution from new particles with mass M_Q. We point out that $\Delta b < 0$: indeed, matter fields can give only negative contributions to the coefficient of the β function, as can be seen from equation 4.12.

Neglecting threshold corrections at M_Q and demanding that the couplings do not have Landau poles below $M_{\rm Pl}$, equation 4.4 gives the constraint:

$$b^{BSM} = b^{SM} + \Delta b \geq -\frac{8\pi^2 + b^{SM}\log\frac{M_Q}{\mu_1}}{g^2(\mu_1)\log\frac{M_{\rm Pl}}{M_Q}} \tag{4.25}$$

where $\mu_1 < M_Q$ In the Standard Model, at scales $\mu > m_t \simeq 173\,\text{GeV}$, the running of the electroweak gauge couplings is determined by the coefficients[7]

$$b_Y^{SM} = -\frac{41}{6}, \quad b_2^{SM} = \frac{19}{6} \tag{4.26}$$

Using the values of $\alpha_{em}(M_Z) \approx 1/128$ and $\sin^2(\theta_W(M_z)) \approx 0.23$ from the PDG Review of Particle Physics [82], we have:

$$\begin{aligned} g_Y^2(M_Z) &= 4\pi\frac{\alpha_{em}(M_Z)}{\cos^2(\theta_W(M_z))} \approx 0.13 \\ g_2^2(M_Z) &= 4\pi\frac{\alpha_{em}(M_Z)}{\sin^2(\theta_W(M_z))} \approx 0.43 \end{aligned}$$

Assuming $M_Q \approx 1\,\text{TeV}$, we obtain:

$$b_Y^{BSM} \geq -14 \quad \Longrightarrow \quad \Delta b_Y \geq -7 \tag{4.27}$$

$$b_2^{BSM} \geq -5.5 \quad \Longrightarrow \quad \Delta b_2 \geq -8.5 \tag{4.28}$$

Contributions from electroweak multiplets and hypercharges

From equation 4.12, for a non-abelian gauge group SU(N) each Weyl fermion transforming in the representation r contributes to the β function

$$\Delta b_2 = -\frac{2}{3}T^{(r)}$$

Using the following group theory identity, valid for $\mathrm{SU}(N)$ representations:

$$C_2^{(r)} \times \dim(r) = (N^2-1)T^{(r)} \tag{4.29}$$

and specialising to the case $N = 2$ for which we know that

$$\dim(r) = 2j+1 \qquad C_2^{(r)} = j(j+1) = \frac{1}{4}(\dim(r)-1)(\dim(r)+1)$$

[7] Here b_Y^{SM} is the one loop beta function coefficient relative to the hypercharge coupling constant. b_2^{SM} is the one loop coefficient relative to the running of the SU(2) electroweak coupling constant.

we derive the relation:

$$T^{(r)} = \frac{1}{12}\dim(r)(\dim(r)-1)(\dim(r)+1) \tag{4.30}$$

For SU(2) doublets and the triplets this gives:

$$T^{(2)} = \frac{1}{2} \qquad\qquad T^{(3)} = 2$$

For an abelian gauge group U(1), a Weyl fermion with hypercharge y contributes as

$$\Delta b_Y = -\frac{2}{3}\,y^2$$

4.3.1 Model with a triplet and two singlets under $\mathbf{SU}(2)_{\mathbf{EW}}$

Let us consider a model in which three of the 5 adjoint Weyl fermions transform as a triplet of the electroweak SU(2) gauge group, with zero hypercharge, and the other two are singlets of the Standard Model. Schematically:

$$\begin{aligned} \text{Gauge group :} \quad & \mathrm{SU(3)_{DC}} \times \mathrm{SU(3)_c} \times \mathrm{SU(2)_{EW}} \times \mathrm{U(1)_Y} \\ \text{Fields :} \quad & (\mathrm{adj}; 1, 3)_0 + (\mathrm{adj}; 1, 1)_0 + (\mathrm{adj}; 1, 1)_0 \end{aligned}$$

Using the notation of reference [1] we denote the triplet with zero hypercharge as V and the singlets as N_1, N_2, calling this model VNN.

For this model $\Delta b_Y = 0$ but $\Delta b_2 = -32/3$, therefore it has a Landau pole near the Grand Unification scale $M_{\mathrm{GUT}} \sim 10^{15}\,\mathrm{GeV}$. Even though the Landau pole is below the Planck mass, we choose to study this model as a relevant example. A UV completion will be needed at the GUT scale, which is anyhow an incredibly high and physically motivated scale.

Using a two components spinor notation, the most general renormalizable lagrangian for this model is given by

$$\begin{aligned} \mathcal{L} = \mathcal{L}_{SM} &- \frac{1}{4}\mathcal{G}^a_{\mu\nu}\mathcal{G}^a_{\mu\nu} - \frac{1}{2\xi}\left(\partial_\mu A^a_\mu\right)^2 + \left(\partial_\mu \bar{c}^a\right)\left(\delta^{ac}\partial_\mu + g_{\mathrm{DC}} f^{abc} A^b_\mu\right)c^c \\ &+ V^\dagger_{a,i} i\bar{\sigma}^\mu D^{ab,ij}_\mu V_{b,j} - \frac{1}{2}M^{ab,ij}_V\left(V_{a,i}V_{b,j} + V^\dagger_{a,i}V^\dagger_{b,j}\right) \\ &+ N^\dagger_{a,p} i\bar{\sigma}^\mu D^{ab,pq}_\mu N_{b,q} - \frac{1}{2}M^{ab,pq}_N\left(N_{a,p}N_{b,q} + N^\dagger_{a,p}N^\dagger_{b,q}\right) \end{aligned} \tag{4.31}$$

where the $a = 1, ..., N^2_{\mathrm{DC}} - 1$ (dark colour), $i = 1, 2, 3$ (electroweak triplet), $p = 1, 2$ (singlets) and the Einstein summation convention is understood. The fields $V_{a,i}$ and $N_{a,p}$ are left-handed Weyl spinor fields[8] and carry a Lorentz index $\alpha = 1, 2$ which is understood and properly contracted[9].

The field strength is defined as usually

$$\mathcal{G}^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_{\mathrm{DC}} f^{abc} A^b_\mu A^c_\nu$$

[8] They are a $\left(\frac{1}{2}, 0\right)$ representation of the Lorentz group. Under a generic rotation of angle θ_i and boost of rapidity β_i, they transform as $V, N \to e^{\frac{1}{2}(i\theta_i\sigma_i + \beta_i\sigma_i)}V, N$.

[9] We use the convention $\psi_1\psi_2 \equiv \psi_1^\alpha\psi_{2\alpha} = \varepsilon^{\alpha\beta}\psi_{1\beta}\psi_{2\alpha}$

The covariant derivative acting on the triplet is

$$D_\mu^{ab,ij} = \delta^{ab}\delta^{ij}\partial_\mu - \delta^{ij} i g_{\rm DC} A_\mu^c \left(T^c_{adj}\right)^{ab} - \delta^{ab} i g_2 W_\mu^k \left(t_V^k\right)^{ij}$$

The covariant derivative acting on the singlet is

$$D_\mu^{ab,pq} = \delta^{ab}\delta^{pq}\partial_\mu - \delta^{pq} i g_{\rm DC} A_\mu^c \left(T^c_{adj}\right)^{ab}$$

All the fermions transform as the adjoint of $\mathrm{SU}(3)_{\rm DC}$; moreover, the SU(2) triplet corresponds to the adjoint representation. Therefore we need the generators of the adjoint representations of SU(3) and SU(2), which can be expressed as

$$\begin{aligned} \left(T^c_{adj}\right)^{ab} &= -if^{cab} = -if^{abc} \\ \left(t_V^k\right)^{ij} &= -i\varepsilon^{kij} = -i\varepsilon^{ijk} \end{aligned}$$

where f^{abc} are the structure constants[10] of SU(3) and ε^{ijk} is the Levi-Civita tensor.

The mass term for the triplet must be invariant under $\mathrm{SU}(3)_{\rm DC}$ and SU(2). For a real representation the most general invariant mass term can be written in a base in which the generators are antisymmetric and real[11] in the following way:

$$M_V^{ab,ij} = M_V \delta^{ab}\delta^{ij}$$

Indeed one has that

$$M_V \longrightarrow e^{i\alpha T^T} M_V e^{i\alpha T} = e^{i\alpha T^T} e^{i\alpha T} M_V = e^{-i\alpha T} e^{i\alpha T} M_V = M_V$$

where $T^T = -T$ follows from the antisymmetry of the generators. We stress that this form of the mass term is consistent with the Fermi-Dirac nature of the fields. Indeed the Lorentz structure is antisymmetric (there is an ε in the contraction of the Lorentz indices), while the mass matrix is symmetric in its indices, giving a antisymmetric mass term.

The most general mass term for the singlets can be written as:

$$M_N^{ab,pq} = \delta^{ab}\begin{pmatrix} M_{11} & M_{12} \\ M_{12} & M_{22}\end{pmatrix}$$

where the diagonal elements correspond to Majorana mass terms and the off-diagonal elements correspond to a Dirac mass term. It is always possible to diagonalise this matrix through an orthogonal transformation that rotates the fields N_p. Therefore we can always redefine the fields in such a way that the mass matrix is

$$M_N^{ab,pq} = \delta^{ab}\begin{pmatrix} M_{N_1} & 0 \\ 0 & M_{N_2}\end{pmatrix}$$

[10] f^{abc} is completely antisymmetric and real [67].

[11] As the one that we are using.

Therefore, we are dealing with a lagrangian describing a triplet of Majorana spinors V with mass M_V transforming as an adjoint representation of SU(2) and two Majorana spinors $N_{1,2}$ of masses M_{N_1} and M_{N_2}, singlets under $\mathcal{G_{SM}}$. Each one has a dark colour index and transforms as the adjoint of $\mathrm{SU}(3)_{\mathrm{DC}}$.

It is possible to rewrite the lagrangian using a four component notation for the Majorana spinors. Defining

$$\Psi_V = \begin{pmatrix} V \\ V^c \end{pmatrix} \qquad \Psi_{N_1} = \begin{pmatrix} N_1 \\ N_1^c \end{pmatrix} \qquad \Psi_{N_2} = \begin{pmatrix} N_2 \\ N_2^c \end{pmatrix}$$

where $V^c = i\sigma^2 V^\dagger = \varepsilon V^\dagger$, the lagrangian becomes:

$$\begin{aligned}\mathcal{L} = \mathcal{L}_{\mathrm{SM}} &- \frac{1}{4}\mathcal{G}^a_{\mu\nu}\mathcal{G}^a_{\mu\nu} - \frac{1}{2\xi}\left(\partial_\mu A^a_\mu\right)^2 + \left(\partial_\mu \bar{c}^a\right)\left(\delta^{ac}\partial_\mu + g_{\mathrm{DC}} f^{abc} A^b_\mu\right)c^c \\ &+ \frac{1}{2}\bar{\Psi}^a_{N_1}\left(i\not{D}^{ab} - M_{N_1}\delta^{ab}\right)\Psi^b_{N_1} + \frac{1}{2}\bar{\Psi}^a_{N_2}\left(i\not{D}^{ab} - M_{N_2}\delta^{ab}\right)\Psi^b_{N_2} \\ &+ \frac{1}{2}\bar{\Psi}^{a,i}_V\left(i\not{D}^{ij,ab} - M_V\delta^{ab}\delta^{ij}\right)\Psi^{b,j}_V\end{aligned} \tag{4.32}$$

The triplet has one electromagnetically neutral component and two components with charge ± 1. Loop corrections split the multiplet, increasing the mass of the charged components and leaving the neutral one as the lightest. The mass splitting is not affected by dark colour interactions and corresponds to the one of an electroweak multiplet. This has been computed in reference [83] and it is of order 150 MeV.

Accidental stability

In this model the Majorana nature of the fermionic fields forbids the presence of U(1) global charges, *i.e.* the Majorana mass term breaks explicitly the global U(1) symmetries associated to each species.

However, the renormalizable lagrangian has three accidental $\mathbb{Z}_2$ symmetries, acting on the corresponding species[12]

$$\begin{aligned} \mathbb{Z}_2^{N_1}: &\qquad \Psi_{N_1} \to -\Psi_{N_1} \\ \mathbb{Z}_2^{N_2}: &\qquad \Psi_{N_2} \to -\Psi_{N_2} \\ \mathbb{Z}_2^{V}: &\qquad \Psi_{V} \to -\Psi_{V} \end{aligned}$$

These symmetries guarantee, for each family, the stability of the lightest state containing an odd number of dark quarks, the gluequark χ.

The three symmetries can be combined into a global $\mathbb{Z}_2^{DC}$ symmetry, which we shall call *dark parity*:

$$\mathbb{Z}_2^{\mathrm{DC}}: \qquad \Psi_{N_1} \to -\Psi_{N_1}, \quad \Psi_{N_2} \to -\Psi_{N_2}, \quad \Psi_V \to -\Psi_V$$

In this way, the stability of the dark matter candidate follows naturally from the renormalizability of the theory, with no further *ad-hoc* symmetry requirement.

[12] The unspecified transformation rules, relative to the other fields, are intended to be the identity.

At the non-renormalizable level, however, the above symmetries can be broken by higher dimensional operators. If we consider our lagrangian as an effective field theory, all the higher dimensional operators compatible with the symmetries of the model should be included. These operators will be suppressed by powers of the cut-off scale of the theory $\Lambda_{\rm UV}$ and give suppressed contributions to low energy observables.

Nevertheless, if they are the only source of the breaking of an accidental symmetry, they give the dominant contribution to the rate of processes violating this symmetry. In order to estimate this rate one should understand what is the dimension of the lowest dimensional operator that induces the breaking. The higher the dimension, the higher the suppression of the effect.

In order to break the $\mathbb{Z}_2^{\rm DC}$ an operator has to include an odd number of dark quarks operators. In addition, it has to be a gauge singlet and a Lorentz scalar.

The lowest dimensional operators that include an odd number of dark quarks and are dark colour singlet are

$$\Psi\mathcal{G}_{\mu\nu}, \qquad \Psi\Psi\Psi$$

and have dimension 7/2 and 9/2 respectively. These are fermionic operators; to have a Lorentz scalar it is then necessary to include an other fermionic field, that should be a singlet of dark colour and should have the right quantum numbers to give an operator that is a Standard Model singlet.

For the triplet V, the lowest dimensional operator that satisfies these conditions is:

$$\frac{1}{\Lambda_{\rm UV}^2} H\sigma^i L_L \sigma^{\mu\nu} V_i \mathcal{G}_{\mu\nu}$$

where H is the Higgs doublet and L_L is the left-handed lepton doublet and σ^i are the Pauli matrices (with electroweak indices).

Similarly, for the singlets N_1 and N_2, the lowest dimensional operator built using only Standard Model fields is:

$$\frac{1}{\Lambda_{\rm UV}^2} H L_L \sigma^{\mu\nu} N_1 \mathcal{G}_{\mu\nu}, \qquad \frac{1}{\Lambda_{\rm UV}^2} H L_L \sigma^{\mu\nu} N_2 \mathcal{G}_{\mu\nu}$$

These operators have dimension 6 and are suppressed by two powers of $\Lambda_{\rm UV}^2$. They induce the decay of the gluequark , for example $\mathcal{X}_1 \to \Phi\ \nu_L$, with an estimated width of order:

$$\Gamma_{\mathcal{X}} \sim \frac{1}{4\pi}\frac{v^2}{\Lambda_{\rm UV}^4} M_{\mathcal{Q}}^3 \sim 10^{-50}\left(\frac{M_{\mathcal{Q}}}{\rm TeV}\right)^3\left(\frac{10^{15}\,\rm GeV}{\Lambda_{\rm UV}}\right)^4 \rm TeV$$

where $v \approx 174\,\rm GeV$ is the Higgs vacuum expectation value. In order to have a cosmologically stable candidate, we should compare its lifetime with the age of the universe $\tau_{\rm univ} \approx 10^{17}\,\rm s$. This translates into the requirement $\Gamma_{\mathcal{X}} < 10^{-44}\,\rm TeV$. For dark quarks of mass $M_{\mathcal{Q}} \sim$ TeV and cut-off $\Lambda_{\rm UV} = M_{\rm GUT} \approx 10^{15}\,\rm GeV$ or higher, the bound is satisfied.

We point out that these operators can also generate neutrino masses through so called type 1 see-saw (singlet) and type 3 see-saw (triplet). Indeed, the composite states $\mathcal{X}_1$ and $\mathcal{X}_2$ have the right quantum numbers to play the role of "right-handed neutrinos" (fermionic singlets), while

χ_V can play the role of the fermionic triplet of the type 3 see-saw.

4.3.2 Model with two doublets and a singlet under $\mathbf{SU(2)_{EW}}$

Let us consider now a model in which the four adjoint Weyl fermions transform as two electroweak doublets, with hypercharges $\pm\frac{1}{2}$, and the fifth is a Standard Model singlet. As before, schematically:

$$\begin{aligned}\text{Gauge group}: &\quad \mathrm{SU}(3)_{\mathrm{DC}} \times \mathrm{SU}(3)_{\mathrm{c}} \times \mathrm{SU}(2)_{\mathrm{EW}} \times \mathrm{U}(1)_{\mathrm{Y}} \\ \text{Fields}: &\quad (\mathrm{adj};1,2)_{-\frac{1}{2}} + (\mathrm{adj};1,2)_{\frac{1}{2}} + (\mathrm{adj};1,1)_0\end{aligned}$$

We denote the two doublets as L_1, L_2 and the singlet as N. We shall refer to this model as the LLN model.

The new fields contribute to the Standard Model β function: $\Delta b_Y = -16/3$ and $\Delta b_2 = -16/3$, therefore the model has no Landau poles below the Planck scale.

Writing as before the most general renormalizable lagrangian for this model in a two components spinor notation we have

$$\begin{aligned}\mathcal{L} = \mathcal{L}_{SM} &- \frac{1}{4}\mathcal{G}^a_{\mu\nu}\mathcal{G}^a_{\mu\nu} - \frac{1}{2\xi}\left(\partial_\mu A^a_\mu\right)^2 + \left(\partial_\mu \bar{c}^a\right)\left(\delta^{ac}\partial_\mu + g_{\mathrm{DC}} f^{abc} A^b_\mu\right)c^c \\ &+ L^\dagger_{1\,a,i} i\bar{\sigma}^\mu D^{ab,ij}_\mu L_{1\,b,j} + L^\dagger_{2\,a,i} i\bar{\sigma}^\mu D^{ab,ij}_\mu L_{2\,b,j} + N^\dagger_a i\bar{\sigma}^\mu D^{ab}_\mu N_b + \mathcal{L}_{\mathrm{mass}}\end{aligned} \tag{4.33}$$

where the covariant derivative acting on the singlet is the same as before, while the one acting on the doublet is

$$D^{ab,ij}_\mu = \delta^{ab}\delta^{ij}\partial_\mu - \delta^{ij} i g_{\mathrm{DC}} A^c_\mu \left(T^c_{adj}\right)^{ab} - \delta^{ab} i g_2 W^k_\mu \left(\sigma^k\right)^{ij} - \delta^{ab}\delta^{ij} i g_Y Y B_\mu$$

with $Y = -1/2$ for L_1 and $Y = 1/2$ for L_2.

The mass term, suppressing the indices which are intended to be properly contracted to give gauge singlets and Lorentz scalars operators, can be written as:

$$\begin{aligned}-\mathcal{L}_{\mathrm{mass}} = &M_L\left(L_1 L_2 + L_2^\dagger L_1^\dagger\right) + \frac{1}{2}M_N\left(NN + N^\dagger N^\dagger\right) \\ &+ y_1 L_1 H N + y_2 L_2 H^c N + y_1^* L_1^\dagger H^\dagger N^\dagger + y_2^* (L_2^c)^\dagger H N^\dagger\end{aligned} \tag{4.34}$$

The Yukawa interactions, after the breaking of the electroweak symmetry, induce a mixing of the neutral components of the doublets L_1 and L_2 with the singlet N. The mass of the charged components of the doublets L_1 and L_2 is not affected by the Yukawa interactions and is given by M_L. The mass term for the three neutral states can be recast in a matrix form

$$-\mathcal{L}_{mass} \supset \frac{1}{2}\begin{pmatrix} L_1^0 & L_2^0 & N\end{pmatrix}\begin{pmatrix} 0 & M_L & y_1 v \\ M_L & 0 & y_2 v \\ y_1 v & y_2 v & M_N\end{pmatrix}\begin{pmatrix} L_1^0 \\ L_2^0 \\ N\end{pmatrix} + h.c.$$

Let us assume for simplicity $y_1 = y_2 = y$. The mass term can be diagonalised by

$$M_{\text{diag}} = U^{\text{T}} M U$$

where U is a 3×3 unitary matrix.

Defining $\Delta M = M_N - M_L$, at the lowest order in the Yukawa couplings the mass term becomes[13]:

$$-\mathcal{L}_{mass} \supset \frac{1}{2}\begin{pmatrix} N_1 & N_2 & N_3 \end{pmatrix}\begin{pmatrix} M_L & 0 & 0 \\ 0 & M_L - 2\frac{(yv)^2}{\Delta M} & 0 \\ 0 & 0 & M_N + 2\frac{(yv)^2}{\Delta M} \end{pmatrix}\begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix} + h.c.$$

where

$$\begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix} = U^\dagger \begin{pmatrix} L_1^0 \\ L_2^0 \\ N \end{pmatrix} = \begin{pmatrix} \frac{-i}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\frac{\sqrt{2}y^* v}{\Delta M} \\ \frac{y^* v}{\Delta M} & \frac{y^* v}{\Delta M} & 1 \end{pmatrix}\begin{pmatrix} L_1^0 \\ L_2^0 \\ N \end{pmatrix} \tag{4.35}$$

The Yukawa interaction with the Higgs boson in the flavour basis is given by

$$\mathcal{L}_{\text{Yuk}} \supset \begin{pmatrix} L_1^0 & L_2^0 & N \end{pmatrix}\begin{pmatrix} 0 & 0 & y \\ 0 & 0 & y \\ y & y & 0 \end{pmatrix}\begin{pmatrix} L_1^0 \\ L_2^0 \\ N \end{pmatrix} h(x) + h.c.$$

Differently from the Standard Model, in this model the Yukawa matrix is not proportional to the mass matrix; therefore, going to the mass basis

$$Y_{\text{mass}} = U^{\text{T}} Y_{\text{flav}} U$$

we get flavour changing interactions mediated by the Higgs boson:

$$Y_{\text{mass}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\frac{4y^2 v}{\Delta M} & \sqrt{2}y \\ 0 & \sqrt{2}y & \frac{4y^2 v}{\Delta M} \end{pmatrix}$$

Similarly, rotating the coupling to the Z bosons we get flavour changing neutral currents in the dark sector

$$U^\dagger \begin{pmatrix} -\frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix} U = \begin{pmatrix} 0 & \frac{i}{2} & \frac{iyv}{\sqrt{2}\Delta M} \\ -\frac{i}{2} & 0 & 0 \\ -\frac{iy^* v}{\sqrt{2}\Delta M} & 0 & 0 \end{pmatrix}$$

If $\Delta M > m_h$ then the Yukawa interaction induces the decay $N_3 \to N_2\, h$ with a decay width

$$\Gamma_{N_3 \to N_2\, h} = \frac{y^2}{8\pi}\sqrt{\frac{\left(M_3^2 - M_2^2 - m_h^2\right)^2 - 4M_2^2 m_h^2}{2M_3^2}}$$

[13]We are assuming $yv \ll \Delta M$.

Similarly, for the case $\Delta M < -m_h$, the Yukawa interaction induces the decay of N_2 into N_3.

In the intermediate range $-m_h < \Delta M < m_h$ the decay can proceed through a virtual Higgs boson decaying to light Standard Model particles in the final state.

The interaction with the Z connects the states N_3 and N_1, inducing the decay of the heavier one of the two.

Accidental stability

The renormalizable lagrangian for this model has some accidental symmetries. Differently from the previous model, the individual $\mathbb{Z}_2^L$ and $\mathbb{Z}_2^N$ are broken explicitly by the Yukawa interactions. However, dark parity is still a valid symmetry, *i.e.* there is a $\mathbb{Z}_2^{\mathrm{DC}}$ symmetry acting only on the fermionic fields of the dark sector.

This ensures the stability of the gluequark at the renormalizable level. At the level of non-renormalizable interactions, we must include all the higher dimensional operators consistent with gauge and Lorentz invariance.

The lowest dimensional operators odd under dark parity and that involves the doublet L_1 or the singlet N have dimension 6 and are respectively

$$\frac{1}{\Lambda_{\mathrm{UV}}^2} L_1 \mathcal{G}_{\mu\nu} \sigma^{\mu\nu} e^c H^c, \qquad \frac{1}{\Lambda_{\mathrm{UV}}^2} N \mathcal{G}_{\mu\nu} \sigma^{\mu\nu} l^L H$$

where e and l_L refer to the right-handed electron and lepton doublet of the Standard Model. These operators induce the decay of the gluequarks, with a decay width that can be estimated, similarly to the case previously analysed, as

$$\Gamma_\chi \sim \frac{1}{4\pi} \frac{v^2}{\Lambda_{\mathrm{UV}}^4} M_{\mathcal{Q}}^3 \sim 10^{-50} \left(\frac{M_{\mathcal{Q}}}{\mathrm{TeV}}\right)^3 \left(\frac{10^{15}\,\mathrm{GeV}}{\Lambda_{\mathrm{UV}}}\right)^4 \mathrm{TeV}$$

This corresponds to a lifetime longer than the age of the Universe for $\Lambda_{\mathrm{UV}} \geq M_{\mathrm{GUT}}$.

However, in this model it is possible to write a dimension 5 operator with an odd number of dark quarks, involving the doublet L_2:

$$\frac{1}{\Lambda_{\mathrm{UV}}} L_2^a \sigma^{\mu\nu} L_L \mathcal{G}_{\mu\nu}^a$$

where L_L is the Standard Model electroweak doublet with hypercharge $-1/2$; the operator can be written only for the L_2 doublet which has hypercharge $+1/2$, .

This operator is suppressed only by one power of the ultraviolet cut-off scale Λ_{UV} and it induces the decay of the neutral mass eigenstates $N_{1,2,3}$, through their mixing with L_2. We can estimate the decay width as

$$\Gamma_{N_i} \sim |\epsilon_i|^2 \frac{1}{4\pi} \frac{1}{\Lambda_{\mathrm{UV}}^2} M_{\mathcal{Q}}^3 \sim |\epsilon_i|^2 10^{-33} \left(\frac{M_{\mathcal{Q}}}{\mathrm{TeV}}\right)^3 \left(\frac{10^{19}\,\mathrm{GeV}}{\Lambda_{\mathrm{UV}}}\right)^4 \mathrm{TeV}$$

where ϵ_i is the mixing of the of the mass eigenstate N_i with the neutral component of the doublet L_2. We have $|\epsilon_{1,2}|^2 = 1/2$ and $|\epsilon_3|^2 = (yv)^2/(\Delta M)^2$ (see equation 4.35).

For typical M_Q and $\Lambda_{\rm UV}$, this corresponds to a lifetime of order $\tau_{N_i} \sim |\epsilon_i|^2 10^6\,$s. For the gluequarks N_1 and N_2 this is much smaller then the age of the universe; these two states are unstable on cosmological scales and cannot be the dark matter candidate.

The state N_3 can be stable on cosmological scales if the Yukawa coupling satisfies the condition

$$y \lesssim 10^{-5} \frac{\Delta M}{1\,\text{TeV}} \tag{4.36}$$

The constraint we have obtained on y is quite strong even if the mass difference between the dark quarks is large, $\Delta M \sim \mathcal{O}(1\,\text{TeV})$. Although this is unpleasant, we note that also in the Standard Model we observe small Yukawa couplings. For instance, the electron has $y_e \sim \mathcal{O}\big(10^{-6}\big)$. Indeed, it is technically natural to have small Yukawa couplings: in the limit of zero Yukawas we recover two additional symmetries: $\mathbb{Z}_2^L$ and $\mathbb{Z}_2^N$, the $\mathbb{Z}_2$ symmetries acting on L-type or N-type dark quarks separately.

We stress that even if we assume $y_1 \neq y_2$, the previous constraint applies to the two Yukawa couplings separately (*i.e.* even if one of the two vanishes). Indeed, we can understand the situation in terms of symmetries: the Dirac mass term couples L_1 and L_2 at order one, so that there is a unique symmetry $\mathbb{Z}_2^L$. If the singlet N is coupled to one of the two doublets through a Yukawa coupling (it is sufficient to have one of the two Yukawas different from zero), the symmetries $\mathbb{Z}_2^L$ and $\mathbb{Z}_2^N$ are no longer valid individually; we are left with a single $\mathbb{Z}_2^{\rm DC}$. At the non-renormalizable level, this symmetry is broken by the dimension 5 operator; a cosmologically stable dark matter candidate can be obtained if the Yukawa couplings satisfy the condition 4.36.

Another possibility is that the coefficient in front of the operator is sufficiently small, with no constraint on the Yukawa coupling. This circumstance would be *technically natural*, since in this limit we recover the $\mathbb{Z}_2^{DC}$ symmetry, but it is somehow contrary to the spirit of *accidental stability*.

Lastly, we point out that this dimension 5 operator is peculiar of models with adjoint fermions. Indeed, in models with fermions transforming as representations different from the adjoint the operator $\Psi\mathcal{G}_{\mu\nu}$ is not a singlet. The lowest dimensional operator with an odd number of dark quarks that is a singlet of dark colour is $\Psi\Psi\Psi$; therefore, non-renormalizable operators violating dark parity can arise only at the level of dimension 6. For comparison, models realising vectorlike confinement have baryonic dark matter candidates whose stability is ensured up to dimension 6 operators.

Custodial symmetry

Custodial symmetry is an approximate global symmetry in the Standard Model that explains, in an elegant and conceptually clear way, the reason why the ρ parameter

$$\rho \equiv \frac{M_W^2}{\cos^2(\theta_{\rm W}) M_Z^2}$$

is so close to unity [84].

Switching off gauge and Yukawa interactions, the Standard Model electroweak sector has an enhanced global symmetry:

$$\mathrm{SO}(4) \simeq \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$$

with the Higgs boson transforming as a fundamental representation $\phi = 4$ of SO(4) or, equivalently a $\mathcal{H} = (2, 2)$ of $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$, where we use the following notation:

$$H = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} \qquad \phi = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{pmatrix} \qquad \mathcal{H} = \begin{pmatrix} H^c & H \end{pmatrix}$$

At the global level the pattern of symmetry breaking induced by the Higgs field acquiring a vacuum expectation value $\langle \phi_3 \rangle = v$ is

$$\mathrm{SO}(4) \to \mathrm{SO}(3)$$

The number of Goldstone bosons is given by the number of broken generators: $6 - 3 = 3$. The three Goldstone bosons transform as a triplet of SO(3).

The gauging of the group $\mathrm{SU}(2)_L$ alone[14] does not break the custodial symmetry. But now, the vacuum expectation value of the Higgs field induces the Higgs mechanism for $\mathrm{SU}(2)_L$; the gauge fields associated to the three broken generators acquire all the same mass M_W.

Switching on the hypercharge gauge group $\mathrm{U}(1)_Y$ induces an explicit breaking of the custodial SO(3). At tree level this gives the following relation

$$M_W = \cos(\theta_W) M_Z$$

This tree level effect due to the breaking of custodial symmetry is reabsorbed in the definition of the ρ parameter so that at tree level one has $\rho = 1$. Further corrections to the ρ parameter must be induced by loop diagrams involving hypercharge insertions or by further source of custodial symmetry breaking.

Let us consider the Yukawa couplings in the Standard Model:

$$\mathcal{L}_{Yuk} = Y_u \bar{Q}_L H^c u_R + Y_d \bar{Q}_L H d_R$$

They induce an explicit breaking of the custodial symmetry for $Y_u \neq Y_d$. However, if we were in the situation $Y_u = Y_d$, we could arrange the right handed quarks in a doublet of $\mathrm{SU}(2)_R$

$$\begin{pmatrix} u_R \\ d_R \end{pmatrix} = (1, 2)$$

and restore custodial symmetry. In particular, one finds that the corrections to the ρ parameter

[14]We set to zero for the moment the gauge coupling associated to the hypercharge, *i.e.* $g_Y = 0$.

are proportional to the strength of symmetry breaking effects

$$\Delta\rho \propto (Y_u - Y_d)$$

Similarly, in the LLN model we just analysed, one should understand if custodial symmetry is preserved or broken by the mass term and by Yukawa interactions 4.34.

The dark sector has an SO(4) global symmetry, that can be viewed as a subgroup of the SU(5) symmetry group that there would be if $M_N = M_L = 0$. We can organise the dark quarks in the following representations of SO(4) = SU(2) × SU(2):

$$\mathbf{L} = \begin{pmatrix} L_1 & L_2 \end{pmatrix} = (2,2) \qquad \mathbf{N} = N = (1,1)$$

The mass term can then be rewritten as

$$M_L\left(L_1 L_2 + L_2^\dagger L_1^\dagger\right) + \frac{1}{2}M_N\left(NN + N^\dagger N^\dagger\right) = \frac{1}{2}M_L L_i^j L_k^l \varepsilon^{ik}\varepsilon_{jl} + \frac{1}{2}M_N NN + h.c.$$

and therefore it preserves the custodial SO(4) = SU(2) × SU(2). The Yukawa interaction preserve the custodial symmetry only if $y_1 = y_2 = y$. Indeed in this case we can write the lagrangian as

$$\frac{1}{2}M_L L_i^j H_k^l \varepsilon^{ik}\varepsilon_{jl}$$

We conclude that also in this case, the corrections to the ρ parameter (and to the closely related $\hat{T}$ parameter) are proportional to the difference of the Yukawa couplings:

$$\Delta\rho \propto \Delta\hat{T} \propto (y_1 - y_2)$$

Chapter 5

Phenomenology

We shall study the phenomenology of the two models described in the previous chapter, with an emphasis on the cosmological history and the constraints deriving from cosmological observations. For each model we build a phase diagram, as a function of the quark mass and confinement scale. As we shall see these models exhibit a very reach phase diagram with many different regimes and physical mechanisms describing their cosmological history. All the calculations outlined in this overview are done for the two models described in chapter 4, which are representative models with and without renormalizable interaction involving the Higgs field in the dark sector.

The different regimes are characterised by the lifetime of the glueballs: if they are stable on cosmological scales they could be a dark matter component. It is then important to evaluate their relic abundance and the phenomenological consequences on cosmological observables.

To identify the parameter space region in which glueballs are stable on cosmological scales, we estimate the lifetime of the glueballs (section 5.1). We use an effective field theory approach, matching to the fundamental theory at the confinement scale. We then discuss what are the phenomenological constraints for the parameter space region in which the glueballs are unstable and decay with a lifetime smaller than the age of the Universe.

Next, we want to evaluate the glueball relic density. As we shall see the glueball relic density is proportional to the entropy ratio between the two sectors at the moment of kinetic decoupling. Therefore, we need to estimate the temperature of kinetic decoupling between the dark and the visible sector (section 5.2). This is the temperature below which the two sectors are no longer in thermal equilibrium and their temperatures evolve independently.

In section 5.3 we compute the glueball relic density. The glueball sector exhibits cannibalism and its thermal history is non-standard. We analyse in detail the relevance of the bounds coming from BBN and CMB observations. In particular, we explore the possibility that the glueballs are a dark matter component, briefly discussing the constraint coming from galaxy formation and from the knowledge of the matter power spectrum. We conclude that, in our models, this possibility is not consistent with the observations, excluding the parameter space region with stable glueballs. However, we point out a possible route to obtain glueball dark matter in a concrete model, without assuming a cosmologically secluded dark sector.

We then consider the parameter space region in which the glueballs are unstable and decay before BBN. The only state of the dark sector which is stable on cosmological scales is the neutral gluequark . In section 5.4 we evaluate its thermal relic abundance. We first compute the annihilation cross section - this is a non-trivial calculation (due to the non-abelian structure of the gauge theory). We then include the non-perturbative corrections due to the Sommerfeld enhancement and compute the relic density through an approximate solution to the Boltzmann equation. At last we compare the results obtained in our model to the ones of other classes of models studied in the literature, pointing out what are the main differences and why this model could be an interesting scenario.

5.1 Glueballs decay

Neglecting higher-dimensional operators generated at the UV cut-off scale, dark quarks interact with the Standard Model through gauge interactions and Yukawa terms if allowed by their quantum numbers. We want to estimate the decay rate of dark glueballs in Standard Model particles. To do so, we take advantage of the separation of scales between M_Q and $\Lambda_{\rm DC}$ ($M_Q \gg \Lambda_{\rm DC}$), proceeding in two steps:

- we integrate out the heavy dark quarks at the scale M_Q. Matching to the effective field theory describing dark gluons interactions with Standard Model particles, we write an effective operator describing interaction between dark gluons and Standard Model particles;
- below the confinement scale $\Lambda_{\rm DC}$ we write an effective lagrangian for the glueballs and their interaction with Standard Model particles.

The resulting vertex is then used to estimate the decay rate of glueballs into Standard Model particles.

To complete the matching procedure, we should know the glueball to vacuum matrix element of the gluonic operator. This parameter must be computed at energies of the order of the glueballs mass and non-perturbative methods are needed. In what follow we shall use lattice results obtained for QCD and extrapolated for use with our model.

The glueball spectrum in QCD has been predicted through lattice calculations [76–78]. The lightest glueball is a state with quantum numbers $J^{PC}(\Phi) = 0^{++}$ and mass $M_\Phi = 1750 \pm 150$ MeV.

Similarly, the glueball to vacuum matrix element of local gluonic operators has been determined numerically. For the scalar operator one has [77, 85]

$$F = \langle 0 | \, g^2 \, \mathrm{Tr}(G_{\mu\nu} G_{\mu\nu}) \, | \Phi \rangle = 15.6 \pm 3.2 \ (\mathrm{GeV})^3$$

These values have been obtained from lattice simulations of a pure gauge SU(3) theory with confinement scale $\Lambda_{\rm QCD} \sim 260$ MeV, corresponding to the confinement scale of the physical Quantum Chromodynamics [86]. Rescaling by $\Lambda_{\rm DC}$ we obtain results which are valid in general for our $\mathrm{SU}(3)_{\mathrm{DC}}$ models. We obtain

$$M_\Phi \sim 7\,\Lambda_{\mathrm{DC}} \tag{5.1a}$$
$$F \sim 10^3\,(\Lambda_{\mathrm{DC}})^3 \tag{5.1b}$$

We shall use these results to estimate the glueball decay rates in the two models of interest.

5.1.1 VNN model

In this model there are no Yukawa interactions in the dark sector, therefore the glueball decay proceeds only through gauge interactions. By inspection, the lowest dimensional local operator generated below M_Q that induces glueball decay has dimension 8 and is generated through a box diagram with a loop of heavy fermions:

$$\mathcal{O}_8 = \left(G^a_{\mu\nu}\right)^2 \left(W^i_{\rho\sigma}\right)^2$$

To estimate the decay width of the glueball induced by the operator $\mathcal{O}_8$ we use an effective field theory approach.

The first step of the matching procedure is done choosing the coefficient of the effective operator $\mathcal{O}_8$ so that the value of the amplitude for the annihilation of two dark gluons in two Standard Model gauge bosons (at energies much smaller than M_Q) is reproduced.

The effective lagrangian at energies below M_Q is:

$$\mathcal{L}_{\mathrm{eff}} \supset c_8 N_{\mathrm{DC}} \frac{g^2_{\mathrm{DC}}}{16\pi^2} g_2^2 \frac{1}{M_Q^4} \left(G^a_{\mu\nu}\right)^2 \left(W^i_{\rho\sigma}\right)^2$$

where c_8 is a dimensionless parameter deriving from the matching condition.

Let us consider the regime $M_\Phi < 2M_{W^\pm}$. In this case the main decay mode is to photons and there is a suppression factor $\sin^2(\theta_W)$. Equivalently, we make the substitution

$$g_2^2 \left(W^i_{\rho\sigma}\right)^2 \longrightarrow e^2 (F_{\mu\nu})^2$$

Below the confinement scale we can write an effective lagrangian for the glueballs: the gluonic scalar operator $\left(G^a_{\mu\nu}\right)^2$ interpolates between the vacuum and the glueball state Φ. Parametrising the matrix element as $F = \langle 0|\, g^2_{\mathrm{DC}} \operatorname{Tr}(G_{\mu\nu}G_{\mu\nu})\,|\Phi\rangle$ we obtain the effective vertex:

$$\mathcal{L}_{\phi\gamma\gamma} = c_8 N_{\mathrm{DC}} \frac{F}{M_Q^4} \frac{e^2}{16\pi^2}\, \Phi (F_{\mu\nu})^2 \tag{5.2}$$

The decay width can then be estimated as

$$\Gamma_{\Phi\to\gamma\gamma} \sim 2\pi (c_8)^2 \alpha^2_{em} \frac{N^2_{\mathrm{DC}}}{(16\pi^2)^2} \frac{1}{M_Q^8} F^2 M_\Phi^3$$

where we have include a factor of 1/2 for identical particles in the final state. This estimate is in agreement, up to the factor c_8, with the results of reference [87].

For $N_{\mathrm{DC}} = 3$, using the numerical values of equation 5.1 taken from lattice simulations, and the value of c_8 computed in reference [87] we obtain:

$$\Gamma_{\Phi\to\gamma\gamma} \sim 2 \cdot 10^{-2}\, \frac{(\Lambda_{\mathrm{DC}})^9}{(M_Q)^8} \tag{5.3}$$

In the regime $M_\Phi > 2M_Z$, we must take into account the decay modes in $W^\pm$ or Z bosons. Three body decays $\Phi \to Wl\nu$, $\Phi \to Zll$ are suppressed by a factor $(M_\Phi/M_W)^4(g_2^2/16\pi^2)$ and we neglect them since we are interested in an order of magnitude estimate. Neglecting phase space factors we have

$$\begin{aligned}\Gamma_{\Phi\to ZZ} &\sim \frac{\cos^4(\theta_W)}{\sin^4(\theta_W)}\Gamma_{\Phi\to\gamma\gamma} \sim 3\cdot 10^{-1}\, \frac{(\Lambda_{\mathrm{DC}})^9}{(M_Q)^8}\\ \Gamma_{\Phi\to WW} &\sim 2\frac{1}{\sin^4(\theta_W)}\Gamma_{\Phi\to\gamma\gamma} \sim 8\cdot 10^{-1}\, \frac{(\Lambda_{\mathrm{DC}})^9}{(M_Q)^8}\end{aligned}$$

5.1.2 LLN model

Differently from the previous case, the model with two electroweak doublets and a singlet, includes also Yukawa couplings with the Higgs field (see equation 4.34). These terms can induce the decay of the glueball through a dimension 6 effective operator

$$\mathcal{O}_6 = \left(G^a_{\mu\nu}\right)^2 H^\dagger H$$

If the glueball has a mass $M_\Phi > 250\,\mathrm{GeV}$, the decay in two Higgs bosons is kinematically allowed. Integrating out the heavy fermions and matching as described above, we arrive at the effective interaction

$$\mathcal{L}_{\Phi\to hh} \sim N_{\mathrm{DC}}\frac{y^2}{16\pi^2}\frac{F}{M_Q^2}\,\Phi h^2$$

giving rise to a decay width

$$\Gamma_{\Phi\to hh} \sim \frac{1}{8\pi}y^4\frac{N_{\mathrm{DC}}^2}{(16\pi^2)^2}\frac{1}{M_Q^4}F^2\frac{1}{M_\Phi}$$

The decay in WW, ZZ give a comparable contribution [88].

If $M_\Phi < 250\,\mathrm{GeV}$, the decay can proceed through a virtual Higgs boson decay to Standard Model particles, with the second Higgs field acquiring a vacuum expectation value v[1].

In order to match to the effective field theory, we need to evaluate the amplitude for this process; this can be factorised as

$$\mathcal{A}(\Phi\to \mathrm{SM}) \sim N_{\mathrm{DC}}\frac{y^2}{16\pi^2}\frac{F}{M_Q^2}\frac{v}{M_\Phi^2 - m_h^2}\mathcal{A}(h^*\to\mathrm{SM})$$

where h^* is the virtual Higgs boson. Since h^* carries all the momentum of the incoming glueball

[1] In the mass range $125\,\mathrm{GeV} < M_\Phi < 250\,\mathrm{GeV}$ there is also the three body decay with an on-shell Higgs boson plus a virtual one decaying to a couple of fermions. However this is suppressed by a factor $M_\Phi^2/(16\pi^2 v^2)$ and gives a subleading contribution.

($Q^2 = M_\Phi^2$), this amplitude is equivalent to the one for the decay of an *Higgs-like* boson with mass M_Φ in Standard Model particles. This amplitude has a strong dependence on M_Φ, especially in the mass range $M_\Phi > 100\,\text{GeV}$. For a detailed study see, for instance, reference [88].

Between 100 GeV and 250 GeV there are various channels that give a comparable width. Near 250 GeV the dominant channel is the decay in a couple of W bosons.

In the mass range $2m_b < M_\Phi < 100\,\text{GeV}$, the dominant decay channel is in a pair of b quarks $h^* \to b\bar{b}$. The effective lagrangian is

$$\mathcal{L}_{\Phi\bar{b}b} \sim N_{\text{DC}} \frac{y^2}{16\pi^2} \frac{F}{M_Q^2} \frac{v}{M_\Phi^2 - m_h^2}\, y_b\, \Phi\bar{b}b$$

giving a decay width

$$\Gamma_{\Phi\to\bar{b}b} \sim \frac{3N_{\text{DC}}^2}{4\pi} \frac{y^4}{(16\pi^2)^2} \frac{F^2}{M_Q^4} \frac{M_\Phi}{\left(M_\Phi^2 - m_h^2\right)^2} m_b^2$$

where colour factors have been taken into account. Our result is in agreement with that of reference [89].

In the range of masses near $2m_b$ there are various thresholds and relevant channels ($\bar{c}c$, $\tau\tau$). Glueballs decay in two gluons should also be considered, becoming relevant for masses in the range $2m_\pi < M_\Phi < 2m_b$.

The decay of the virtual Higgs boson in gluons proceeds at one-loop and in order to estimate its amplitude we use again an effective field theory approach, integrating out the quarks heavier than M_Φ. The coefficient of the effective operator inducing the decay of the Higgs can be estimated using Naive Dimensional Analysis [90–92]:

$$\mathcal{L}_{h^*gg} \sim \sum_i N_{\text{c}} \frac{y_i}{m_i} \frac{g_s^2}{16\pi^2} h^* \left(G_{\mu\nu}^a\right)^2$$

where the sum runs on all the quarks heavier than M_Φ. The powers of the Yukawa coupling y_i and of the mass m_i are dictated by NDA, and the loop factor $\frac{g_s^2}{16\pi^2}$ account for the fact that the coupling arises only at one-loop level. Using the relation $m_i = v y_i$ we arrive at

$$\mathcal{L}_{h^*gg} \sim \sum_i N_{\text{c}} \frac{1}{v} \frac{g_s^2}{16\pi^2} h^* \left(G_{\mu\nu}^a\right)^2$$

We see that all the quarks heavier than the glueball contribute the same amount to the decay amplitude in gluons, as a result of the cancellation between the suppression $\frac{1}{m_i}$ and the enhancement y_i for each heavy quark. This is the so called *non-decoupling effect*, well-known in Higgs physics. Therefore, from the sum we get a factor N_{hq} which corresponds to the number of quarks heavier than M_Φ.

We arrive at the effective lagrangian

$$\mathcal{L}_{\Phi gg} \sim 3N_{\text{DC}} N_{hq} \frac{y^2}{16\pi^2} \frac{g_s^2}{16\pi^2} \frac{F}{M_Q^2} \frac{1}{M_\Phi^2 - m_h^2}\, \Phi G_{\mu\nu} G_{\mu\nu}$$

which gives the decay width

$$\Gamma_{\Phi\to gg} \sim \frac{9N_{\rm DC}^2 N_{hq}}{4\pi}\frac{y^4}{(16\pi^2)^2}\frac{g_s^4}{(16\pi^2)^2}\frac{F^2}{M_{\mathcal{Q}}^4}\frac{M_\Phi^3}{\left(M_\Phi^2-m_h^2\right)^2}$$

The decay into photons, in addition to the contribution given by the operator $\mathcal{O}_8$ (equation 5.2), gets a contribution mediated by the Higgs field through the operator $\mathcal{O}_6$. This can be estimated similarly to what we just did for the decay into gluons, including an additional contribution due to a loop with W bosons.

We obtain the effective lagrangian:

$$\mathcal{L}_{\Phi\gamma\gamma} \sim \left(N_{\rm DC}\frac{F}{M_{\mathcal{Q}}^4}\frac{e^2}{16\pi^2} + N_{\rm DC}\sum_i \frac{y^2}{16\pi^2}\frac{Q_i^2 e^2}{16\pi^2}\frac{F}{M_{\mathcal{Q}}^2}\frac{1}{M_\Phi^2-m_h^2}\right)\Phi F_{\mu\nu}F_{\mu\nu} \tag{5.4}$$

where the sum runs on the fermions heavier than M_Φ (both leptons and quarks[2]) and on the W boson.

For Yukawa couplings of order $y \sim 10^{-5}$ or lower, required in order to have cosmologically stable relics, the contribution from the dimension 8 operator $\mathcal{O}_8$ discussed in the previous section dominates, and the glueballs have a similar lifetime to that discussed in the previous section.

5.1.3 Constraints on glueballs decay

Cosmological and astrophysical observations give tight constraints on the decay of long lived thermal relics, that exclude a portion of the parameter space of the models we are considering.

We can identify three different regimes:

- the glueballs are stable on cosmological scales, corresponding to the condition $\tau_\Phi > \tau_{\rm univ}$. We shall consider this scenario in the following two section. In this case we need to evaluate the glueballs relic density and see if they can be a dark matter component and if their presence as thermal relics is compatible with cosmological constraints (number of relativistic degrees of freedom at the epoch of BBN and CMB) and astrophysical observations (structure formation).
- the glueballs are long-lived and decay during or after BBN. This possibility is strongly constrained as we shall analyse in this paragraph.
- the glueballs decay quickly, *i.e.* before the onset of the Big Bang Nucleosynthesis . In order to be consistent with the observations, a lifetime $\tau_\Phi < 1\,\mathrm{s} \Rightarrow \Gamma_\Phi > 10^{-27}\,\mathrm{TeV}$ is required. This situation is basically unconstrained since the earliest cosmological data correspond to the epoch of BBN.

The region in which the glueballs are not cosmologically stable corresponds to the condition

$$\tau_\Phi < \tau_{\rm univ} = 10^{17}\,\mathrm{s} \Rightarrow \Gamma_\Phi > 10^{-44}\,\mathrm{TeV}$$

[2]The colour multiplicity factor $N_{\rm c}$ must be included for each quark, but not for leptons.

In our models, for dark quarks with mass $M_{\mathcal{Q}} \gtrsim 1\,\mathrm{TeV}$, the calculation of the previous paragraph tells us that this condition is realised for confinement scales $\Lambda_{\rm DC} \gtrsim 35\,\mathrm{MeV}$. Therefore, in the regime in which the glueballs are long-lived decaying relics, they are already confined at the epoch of Big Bang Nucleosynthesis $T_{\rm BBN} \approx 1\,\mathrm{MeV}$.

The limits on the number of relativistic degrees of freedom at the epoch of BBN do not apply in this case, but primordial light-elements abundances, predicted by BBN, give other strong constraints on relics decaying electromagnetically and hadronically [93]. In our case, cosmologically unstable glueballs have masses $M_\Phi \sim 7\Lambda_{\rm DC} \gtrsim 250\,\mathrm{MeV}$ and can thus decay both electromagnetically and hadronically (with a smaller branching ratio). The bounds inferred from BBN observations depend on the number density of glueballs and are expressed in terms of the relic abundance the decaying relics would have if they were stable [93]. In the mass range we are considering, the relic abundance of glueballs would be very large $\Omega_\Phi h^2 \gtrsim 5 \cdot 10^6$, as we will see in section 5.3; therefore, the strongest constraint of reference [93] applies, excluding the parameter space region in which glueballs have a lifetime $1\,\mathrm{s} < \tau_\Phi < 10^{12}\,\mathrm{s}$.

An independent bound on long lived glueballs comes from observations of the diffuse gamma rays spectrum [94]. For the mass range and would-be relic abundance of interest, the data exclude the region $10^{12}\,\mathrm{s} < \tau < 10^{17}\,\mathrm{s}$, closing the window in the parameter space with long-lived unstable glueballs.

Summarising, these observations rule out the parameter space region in which glueballs are long-lived, *i.e.* have a lifetime $1\,\mathrm{s} < \tau_\Phi < 10^{17}\,\mathrm{s}$.

5.2 Kinetic decoupling

At high temperatures, in the early universe, the dark sector and the Standard Model sector are in thermal equilibrium. Elastic scattering processes involving particles of both sectors keep the two in equilibrium until the rate of these interactions becomes smaller than the Hubble parameter, that is the expansion rate of the universe. This moment in the thermal history of the universe is called *kinetic decoupling.*

It is important to differentiate kinetic decoupling from *chemical decoupling.* This corresponds to the moment at which number changing processes (such as dark matter annihilation $\mathcal{Q}\mathcal{Q} \to ff$, where f is a Standard Model fermion) cease to be efficient. Chemical decoupling is usually involved in the calculation of the thermal relic density of dark matter candidates, as we shall do in section 5.4. However here we are addressing a different issue: we want to estimate the temperature at which the two sectors are no longer in thermal equilibrium. As we shall see, thermal equilibrium can last much longer than chemical equilibrium.

The crucial difference is in the observation that the rate for a number changing process $\mathcal{Q}\mathcal{Q} \to ff$ is $\sigma v n_{\rm non-rel}$, where $n_{\rm non-rel}$ is the density of targets $\mathcal{Q}$ which is suppressed by a Boltzmann factor, while elastic processes such as $\mathcal{Q}f \to \mathcal{Q}f$ have a rate $\sigma v\, n_{\rm rel}$ with no suppression, since now the density of targets refers to light Standard Model fermions ($n_{\rm rel} \sim T^3$). Therefore, we expect $T_{kd} \ll T_{\rm chem} \sim M_{\mathcal{Q}}/20$. Indeed we shall find that the kinetic decoupling temperature is of order $T_{kd} \sim 100\,\mathrm{MeV}$. We note that the region with cosmologically stable glueballs corresponds to confinement scales smaller than the kinetic decoupling temperature; therefore, in our calculation we consider deconfined dark quarks and not gluequarks.

There are different processes that can contribute to the elastic cross section of dark sector and Standard Model particles. Since, as we have just anticipated, kinetic decoupling temperature is small, we consider only light Standard Model particles in the final state. At tree level, charged particles in the dark sector can interact through electromagnetic and weak interactions, while the neutral ones interact through weak interactions mediated by the Z boson. Interactions involving the Higgs boson (present in the LLN model) are suppressed by the Yukawa couplings of light fermions and are therefore negligible.

The dark quark corresponding to the neutral component of an electroweak multiplet has an induced magnetic dipole moment. However, this is a one-loop effect involving electroweak interactions. We can estimate the magnetic moment as

$$\mu_{\mathcal{Q}^0} \approx \frac{g_2^2}{M_W^2}\frac{e}{16\pi^2}M_{\mathcal{Q}^0} = \frac{G_F}{\sqrt{2}}\frac{M_{\mathcal{Q}^0}m_e}{\pi^2}\mu_B \Longrightarrow \mu_{\mathcal{Q}^0} \approx 4\cdot 10^{-7}\left(\frac{M_{\mathcal{Q}^0}}{1\,\text{TeV}}\right)\mu_B$$

where $\mu_B = e\hbar/2m_e c$ is the Bohr magneton. The electromagnetic scattering scattering cross section induced by this interaction is subdominant with respect to the weak interaction (in the relevant temperature regime) and therefore we neglect it.

At temperatures lower than the electroweak scale the two relevant processes have respectively a cross section:

$$\sigma_{\text{weak}}(\mathcal{Q}f \to \mathcal{Q}f) \sim \frac{1}{4\pi}G_F^2T^2 \qquad\qquad \sigma_{\text{em}}\big(\mathcal{Q}^\pm f \to \mathcal{Q}^\pm f\big) \sim 4\pi\frac{\alpha_{\text{em}}^2}{T^2} \tag{5.5}$$

For typical dark quark masses $M_{\mathcal{Q}} \sim \mathcal{O}(1)$TeV, the weak cross section gives the leading effect for temperatures $T \geq 100\,\text{MeV}$.

At lower temperatures the electromagnetic interaction of the charged relics would dominate. At temperatures higher then the mass difference $M_{\mathcal{Q}^\pm} - M_{\mathcal{Q}^0}$, the two species are both relevant and electromagnetic interactions can be efficient in guaranteeing thermal equilibrium. However, the charged component $\mathcal{Q}^\pm$ is expected to be heavier then the neutral candidate $\mathcal{Q}^0$ and to decay into the neutral one through weak interactions. At temperatures $T < (M_{\mathcal{Q}^\pm} - M_{\mathcal{Q}^0})$ the density of charged species drops and only the weak interactions of $\mathcal{Q}^0$ can be efficient, with cross section $\sigma_{\text{weak}} \sim G_F^2T^2/4\pi$.

In our models, as we discussed in section 4.3, typically the mass difference of the charged and neutral component of an electroweak multiplet is $(M_{\mathcal{Q}^\pm} - M_{\mathcal{Q}^0}) \sim 150\,\text{MeV} > 100\,\text{MeV}$, therefore we can consider the weak cross section σ_{weak} as the relevant one in determining the kinetic decoupling.

To estimate the kinetic decoupling temperature we proceed as follows (our discussion is based on the one in reference [95]; for a more rigorous treatment see [96]).

We need to compare the rate at which elastic scattering reactions are efficient in establishing kinetic equilibrium with the Hubble expansion rate.

Let us consider two gases at the same temperature T: one of relativistic particles (standard model photons and light fermions) with typical energy and momentum T and one of non-relativistic (*cold*) dark quarks[3] with typical momentum $p \sim \sqrt{M_Q T}$, given by the equipartition theorem.

The typical momentum transfer per collision is $\delta p \sim T$. The relative momentum transfer in a single elastic scattering event is then, in average

$$\frac{\delta p}{p} \sim \left(\frac{T}{M_Q}\right)^{\frac{1}{2}} \ll 1$$

In order to keep the dark quarks in thermal equilibrium, we need an order one relative change in momentum $\delta p \sim p$. Therefore, a single scattering process is not sufficient and we need a large number of interactions. This results in a suppression of the effective rate of elastic collisions that give kinetic equilibrium.

Since thermal equilibrium is a stochastic process, after $N_{\rm coll}$ collisions the momentum will be changed by

$$\left(\frac{\delta p}{p}\right)_{\rm TOT} \sim \left(\frac{\delta p}{p}\right)(N_{\rm coll})^{\frac{1}{2}}$$

Accordingly, we can estimate the average number of collisions needed to establish kinetic equilibrium as

$$N_{\rm coll} \sim \frac{M_Q}{T}$$

Let us assume that we are at temperatures $T > 100\,\mathrm{MeV}$, so that the elastic cross section is $\sigma_{\rm weak}$. Comparing the rate of effective collisions with the Hubble rate we obtain the temperature of kinetic decoupling

$$n_{\rm rel}\,\sigma_t c\,\frac{1}{N_{\rm coll}} \sim H$$

Using the Friedmann's equation in the radiation dominated epoch

$$H^2 = \frac{8\pi G_N}{3}\rho \Rightarrow H \approx \frac{T^2}{M_{\rm Pl}}$$

we have

$$T^3\,\frac{1}{4\pi}G_F^2 T^2\,\frac{T}{M_Q} \sim \frac{T^2}{M_{\rm Pl}}$$

We arrive at the following order of magnitude estimate for the kinetic decoupling temperature

$$T_{kd} \sim 50\,\mathrm{MeV}\left(\frac{M_Q}{1\,\mathrm{TeV}}\right)^{\frac{1}{4}}$$

Therefore we can conclude that

$$T_{kd} \sim \mathcal{O}(100)\,\mathrm{MeV}$$

Below this scale, the two sectors are no longer in thermal equilibrium and their temperatures evolve independently. There is, however, a caveat to this conclusion: after confinement, in

[3]We neglect the dark gluons thermal bath, since at low energies their interactions with standard model particles proceed through irrelevant (*i.e.* higher dimensional) operators.

the dark sector the asymptotic states are glueballs and gluequarks. χ is stable thanks to an accidental symmetry as discussed in section 4.3, whereas glueballs can decay to Standard Model particles. If the decay is fast, what happens is that the entropy density of the dark gluons at the time of confinement is completely transferred to the radiation in the standard sector. This results in a reheating, *i.e.* an increase of the temperature and of the radiation energy density in the standard model sector.

5.3 Glueballs relic density

We shall calculate the relic density of glueballs in the region of the parameter space in which the glueballs are stable on cosmological scales (as described in section 5.1).

As the universe cools down, the dark sector and the Standard Model go out of thermal equilibrium at a temperature $T_{kd} \sim 100\,\text{MeV}$, as described in the previous section. In the following all the primed symbols will refer to quantities relative to the dark sector, while unprimed symbols will refer to Standard Model quantities (*i.e.* T' is the temperature of the dark sector and T is the temperature of the Standard Model sector).

After kinetic decoupling the temperatures in the two sectors evolve independently and the entropy is separately conserved in each sector.

In particular, the ratio of the comoving entropies in the two sectors has a constant value

$$\xi = \frac{s}{s'} = \text{constant} \tag{5.6}$$

In our model the dark sector corresponds to an $\mathrm{SU}(3)_{\mathrm{DC}}$ gauge theory with 5 adjoint fermions and it is in thermal equilibrium with the baryonic thermal bath until T_{kd}. At this temperature, the entropy density in the Standard Model sector [97] can be expressed as a function of the effective number degrees of freedom g (a function of the temperature), parametrising the contribution of relativistic and non-relativistic species:

$$s = \frac{2\pi^2}{45} g T^3$$

If $T_{kd} > T'_{\text{conf}} \approx \Lambda_{\text{DC}}$, the dark sector is unconfined at kinetic decoupling and its entropy is dominated by the relativistic degrees of freedom (the dark gluons), which correspond to $g'_{kd} = 2(N_{\text{DC}}^2 - 1) = 16$.

The entropy ratio is then fixed at this temperature by

$$\xi = \frac{g_{kd} T_{kd}^3}{g'_{kd} T'^3_{kd}} = \frac{g_{kd}}{g'_{kd}} \tag{5.7}$$

Since the kinetic decoupling temperature $T_{kd} \sim \mathcal{O}(100)\,\text{MeV}$ is near the temperature at which the QCD phase transitions undergoes, the number of effective relativistic degrees of freedom at kinetic decoupling has a big uncertainty, being in the range $15 \leq g_{kd} \leq 65$. This gives a ξ parameter in the range

$$1 \lesssim \xi \lesssim 4 \tag{5.8}$$

At temperatures $T'_{\rm conf} < T' < T_{kd}$ both the dark sector and the Standard Model entropies are dominated by radiation. Entropy conservation then gives

$$s \sim gT^3 \propto \frac{1}{a^3} \tag{5.9}$$

and similarly for the dark sector. The number of relativistic degrees of freedom in the dark sector g' does not change until confinement and so $T' \propto \frac{1}{a}$.

At the dark confinement scale, the gluons confine in glueballs. Energy is conserved and the energy density of relativistic gluons gets converted in energy density of glueballs. Therefore we have

$$\rho'_{\Phi,\rm conf} = \frac{\pi^2}{30} 2\big(N_{\rm DC}^2 - 1\big) T'^4_{\rm conf} = \frac{8}{15}\pi^2 T'^4_{\rm conf} \tag{5.10}$$

After the confinement transition, the dark sector is made up of non relativistic particles only: glueballs and gluequark .

If the glueballs are stable on cosmological scales, it is important to evaluate if they can account for a part of the dark matter and what are the bounds from cosmological observations. We shall now evaluate their relic density and discuss if there are regions of the parameter space with stable glueballs that are phenomenologically viable.

5.3.1 No number changing interactions

Let us compute first the relic density of glueballs neglecting number changing interactions in the glueball sector.

$$\begin{aligned} \rho'_{\Phi,0} &= M_\Phi\, n'_0 = M_\Phi\, n'_{\rm conf} \left(\frac{a_{\rm conf}}{a_0}\right)^3 = \rho'_{\Phi,\rm conf} \left(\frac{a_{\rm conf}}{a_0}\right)^3 \\ &= \rho'_{\Phi,\rm conf} \frac{s_0}{s_{\rm conf}} = \rho'_{\Phi,\rm conf} \frac{s_0}{s'_{\rm conf}\,\xi} \end{aligned} \tag{5.11}$$

Using the principles of thermodynamics one can show (see for example [97]) that in the expanding Universe, the entropy density is given by

$$s = \frac{\rho + P - \mu n}{T} \tag{5.12}$$

where P and μ denote the pressure and the chemical potential respectively. At the confinement we have $\mu, P \approx 0$ and therefore

$$\frac{\rho'_{\Phi,\rm conf}}{s'_{\rm conf}} = T'_{\rm conf}$$

Using this relation we finally arrive at

$$\rho'_{\Phi,0} = s_0 \frac{T'_{\rm conf}}{\xi} \tag{5.13}$$

In this regime dark matter is non relativistic and its temperature evolve as $T' \propto \frac{1}{a^2}$.

5.3.2 Cannibalism in the glueball sector

The glueball sector can experience a phase of so called *cannibalism* if number changing reactions in the dark sector keep the glueballs in chemical equilibrium with vanishing chemical potential.

This scenario is different from the standard cold dark matter paradigm and has been studied for the first time in [98] and reconsidered recently in a different perspective in [99,100] and [101,102]. In the usual cold dark matter scenario, the number changing reactions involve the annihilation of dark matter in Standard Model particles and the number of dark matter particles per comoving volume remains constant when this interaction becomes inefficient. This happens when the dark matter is already non relativistic (usually $T_{fo} \sim M_\chi/25$) but the two sectors are still in thermal equilibrium.

In the scenario with cannibalism, what happens is that number changing interactions in the dark sector are efficient even after kinetic decoupling has occurred. Since the two sectors are out of equilibrium, their temperatures evolve independently and their entropies per comoving volume are separately conserved.

When a number changing reaction occurs (for instance a process $3 \to 2$), the energy stored in the rest mass of a glueball is redistributed as kinetic energy, reheating the dark sector. As a result, the dark sector temperature decreases only logarithmically as a function of the scale factor and the glueball energy density has an unusual scale dependence with respect to the standard cold dark matter.

Number changing interactions are usually expected to be present in the glueball sector of a confining gauge theory. Writing an effective action for the glueballs below the confinement scale, Naive Dimensional Analysis [90–92] gives

$$\mathcal{L}_\Phi = \frac{1}{2}(\partial_\mu \Phi)^2 - \frac{M_\Phi^2}{2}\Phi^2 + \frac{c_3}{3!}\frac{4\pi}{\sqrt{N_{\rm DC}}}M_\Phi \Phi^3 + \frac{c_4}{4!}\frac{16\pi^2}{N_{\rm DC}}\Phi^4 + \cdots \tag{5.14}$$

where the coefficients c_i are expected to be $\mathcal{O}(1)$ factors and we assumed that in the non-perturbative region $T' < \Lambda_{\rm DC}$ the theory is in a strong coupling regime : $g_{\rm DC} \sim \frac{4\pi}{\sqrt{N_{\rm DC}}}$.

We shall now derive the scaling law of temperature and energy density during the cannibal phase and the relic density of stable glueballs.

The confinement temperature is $T'_{\rm conf} \sim \Lambda_{\rm DC} < M_\Phi$, glueballs are non relativistic when they are formed. When number changing interactions are efficient, glueballs number density is given by the thermal expression

$$n' = g'\left(\frac{M_\Phi T'}{2\pi}\right)^{\frac{3}{2}} e^{-\frac{M_\Phi}{T'}} \tag{5.15}$$

Moreover, the chemical potential is zero during this phase and the pressure is zero for non relativistic particles, so from equation 5.12 we obtain:

$$s' = \frac{M_\Phi n'}{T'} = g'\frac{M_\Phi}{T'}\left(\frac{M_\Phi T'}{2\pi}\right)^{\frac{3}{2}} e^{-\frac{M_\Phi}{T'}} \tag{5.16}$$

Entropy conservation then gives:

$$s'a^3 = \text{const} \Longrightarrow \bar{a}^3 \equiv \frac{(2\pi)^{\frac{3}{2}}}{g' M_\Phi^3} s' a^3 = a^3 \left(\frac{T'}{M_\Phi}\right)^{\frac{1}{2}} e^{-\frac{M_\Phi}{T'}}$$

where $\bar{a}$ is a dimensionless constant. Neglecting the polynomial dependence on the temperature we arrive at the scaling law:

$$\frac{T'}{M_\Phi} \sim \frac{1}{3\log\left(\frac{a}{\bar{a}}\right)} \Longleftrightarrow \frac{a}{\bar{a}} \sim e^{\frac{M_\Phi}{3T'}} \tag{5.17}$$

Again using equation 5.12 and entropy conservation we arrive at the scaling law for glueball energy density during the phase with cannibalism

$$s' = \frac{\rho'}{T'} \propto \frac{1}{a^3} \Longrightarrow \rho' = s'T' \propto \frac{a^3}{\log\left(\frac{a}{\bar{a}}\right)} \tag{5.18}$$

The phase with cannibalism ends when the rate of number changing interactions in the dark sector becomes smaller than the Hubble rate and chemical equilibrium is lost. This process is known as chemical decoupling, and after it occurs the number of glueballs per comoving volume is fixed.

The glueball density at chemical decoupling can be computed as

$$\rho'_{\Phi,d} = \rho'_{\Phi,\text{conf}} \left(\frac{a_{\text{conf}}}{a_d}\right)^3 \frac{\log\left(\frac{a_{\text{conf}}}{\bar{a}}\right)}{\log\left(\frac{a_d}{\bar{a}}\right)}$$

where a_{conf} is the scale factor at dark confinement and a_d is the scale factor at chemical decoupling.

The present day relic density can then be obtained by rescaling (the number of glueballs per comoving volume is fixed)

$$\rho'_{\Phi,0} = \rho'_{\Phi,\text{conf}} \left(\frac{a_{\text{conf}}}{a_d}\right)^3 \frac{\log\left(\frac{a_{\text{conf}}}{\bar{a}}\right)}{\log\left(\frac{a_d}{\bar{a}}\right)} \left(\frac{a_d}{a_0}\right)^3 = \rho'_{\Phi,\text{conf}} \left(\frac{a_{\text{conf}}}{a_0}\right)^3 \frac{\log\left(\frac{a_{\text{conf}}}{\bar{a}}\right)}{\log\left(\frac{a_d}{\bar{a}}\right)}$$

Using the scaling law for the dark sector temperature (equation 5.17) we arrive at

$$\rho'_{\Phi,0} = \rho'_{\Phi,\text{conf}} \left(\frac{a_{\text{conf}}}{a_0}\right)^3 \frac{T'_d}{T'_{\text{conf}}} \tag{5.19}$$

Comparing with equations 5.11 and 5.13 we obtain the expression

$$\rho'_{\Phi,0}(\text{cannibalism}) = \rho'_{\Phi,0}(\text{no cannibalism}) \frac{T'_d}{T'_{\text{conf}}} = s_0 \frac{T'_{\text{conf}}}{\xi} \frac{T'_d}{T'_{\text{conf}}} = s_0 \frac{T'_d}{\xi} \tag{5.20}$$

We stress here that even though the dark sector cools down more slowly than in the case without cannibalism, the glueball energy density is smaller. This fact can be understood intuitively: for a non relativistic species, the energy density is given by the number density times the mass of the particle. Number changing reactions deplete the number of glueballs, while the increased kinetic energy is not enough to bring up the temperature of the dark sector that keeps cooling down due to the Hubble expansion, even though more slowly. The net result is a lower relic density for the glueballs.

The ratio between mass and temperature at chemical decoupling has been computed in [98]. As for the usual freeze-out of cold dark matter, it depends only logarithmically on the strength of the interaction and typically one has $\frac{M_\Phi}{T'_d} \sim \mathcal{O}(20-40)$.

Multiplying and dividing by M_Φ we can write the glueball relic density as a function of the glueball mass

$$\rho'_{\Phi,0} = s_0 \frac{M_\Phi}{\xi} \left(\frac{T'_d}{M_\Phi}\right)$$

The Standard Model sector entropy today is dominated by the CMB radiation that is well measured by the Planck satellite and is given by $s_0 = 2.9 \times 10^3\,\mathrm{cm}^{-3}$. Dividing the relic density by the critical density[4] $\rho_{\text{crit}} \equiv \frac{3H_0^2}{8\pi G} = 10^{-5}\,h^2\,\mathrm{GeV\ cm}^{-3}$ we obtain the present day density parameter for the glueballs

$$\Omega'_{\Phi,0}h^2 = 0.3\,\frac{M_\Phi}{\xi\,\mathrm{eV}} \left(\frac{T'_d}{M_\Phi}\right) \tag{5.21}$$

Validity of the relic density calculation

The calculation of the glueball relic density presented in the previous section relies on the assumption that the glueballs have enough time to cool down and arrive at the decoupling temperature. However, since the temperature dependence on the scale factor is only logarithmic in the glueball sector, we have to verify if this is the case.

We consider the logarithmic evolution of the dark sector temperature in the era in which number changing interactions are efficient and find what would be the present day temperature if there were no decoupling: $T'_{\log}(a=1)$. In order for the decoupling to have occurred, this temperature must be smaller than the decoupling temperature: $T'_{\log}(a=1) < T'_d$.

$$T'_{\log}(a) \sim \frac{M_\Phi}{3\log\left(\frac{a}{\bar{a}}\right)}$$

At confinement we have:

$$T'_{\log}(a_{\text{conf}}) \sim \Lambda_{\text{DC}} \approx \frac{M_\Phi}{7} \Rightarrow 3\log\left(\frac{a_{\text{conf}}}{\bar{a}}\right) = 7 \Rightarrow a_{\text{conf}} = \bar{a}e^{\frac{7}{3}}$$

It is easy to relate the temperatures in the two sectors at the moment in which dark confinement occurs:

$$\frac{g_{kd}T_{kd}^3}{g_{\text{conf}}T_{\text{conf}}^3} = \frac{a_{\text{conf}}^3}{a_{kd}^3} = \frac{T'_{kd}}{T'^3_{\text{conf}}}$$

from which we obtain

$$T'_{\text{conf}} = \left(\frac{g_{\text{conf}}}{g_{kd}}\right)^{\frac{1}{3}} T_{\text{conf}} \approx T_{\text{conf}}$$

[4] h is the reduced Hubble constant: $H_0 \equiv 100\,h\,\mathrm{km\,s}^{-1}\ \mathrm{Mpc}^{-1}$

We can then derive an expression for the scale factor at confinement

$$T_{\rm conf} \approx T'_{\rm conf} \sim \frac{M_\Phi}{7} \Rightarrow \frac{1}{a_{\rm conf}} \sim \frac{M_\Phi}{7\,T_{\rm CMB}}$$

Putting all together we arrive at

$$T'_{\log}(a=1) = \frac{M_\Phi}{3\log\left(\frac{1}{a_{\rm conf}\,e^{-\frac{7}{3}}}\right)} = \frac{M_\Phi}{3\log\left(\frac{1}{a_{\rm conf}}\right)+7} = \frac{M_\Phi}{3\log\left(\frac{M_\Phi}{7\,T_{\rm CMB}}\right)+7} \tag{5.22}$$

For a typical decoupling temperature $T'_d \sim \frac{M_\Phi}{30}$ we have the inequality

$$T'_{\log}(a=1) < T'_d \Rightarrow \frac{M_\Phi}{3\log\left(\frac{M_\Phi}{7T_{\rm CMB}}\right)+7} < \frac{M_\Phi}{30} \Rightarrow M_\Phi > 7e^8 T_{\rm CMB}$$

Using the CMB temperature $T_{\rm CMB} = 2.73\,\mathrm{K} = 0.25\,\mathrm{meV}$ we find that the result 5.21 is valid only for cosmologically stable glueballs of mass

$$M_\Phi \gtrsim 5\,\mathrm{eV} \tag{5.23}$$

For glueballs of lower mass, the equation for the relic density must be modified replacing the decoupling temperature with the present day dark sector temperature

$$\Omega'_{\Phi,0}h^2 = 0.3\,\frac{M_\Phi}{\xi\,\mathrm{eV}}\left(\frac{T'_{\log}(a=1)}{M_\Phi}\right) = 0.3\,\frac{M_\Phi}{\xi\,\mathrm{eV}}\left(\frac{1}{3\log\left(\frac{M_\Phi}{7\,T_{\rm CMB}}\right)+7}\right) \tag{5.24}$$

This formula for the relic density is valid for glueball masses such that confinement has occurred before today, that is $T_{\rm conf} > T_{\rm CMB}$. The range of validity is

$$M_\Phi \sim 7\Lambda_{\rm DC} \sim 7\,T'_{\rm conf} = 7\left(\frac{g_{\rm conf}}{g_{kd}}\right)^{\frac{1}{3}} T_{\rm conf} \gtrsim 2\,T_{\rm conf} > 2\,T_{\rm CMB} \Longrightarrow M_\Phi \geq 0.5\,\mathrm{meV}$$

where we used the estimate $\left(\frac{g_{\rm conf}}{g_{kd}}\right)^{\frac{1}{3}} \gtrsim \frac{1}{3.5}$.

If $M_\Phi < 0.5\,\mathrm{meV}$ the dark sector temperature today is still higher then the confinement temperature and the dark sector is in the deconfined phase. The energy density of dark gluons is then given by the standard relation for relativistic radiation

$$\rho'_{\rm Dgluons} = \frac{\pi^2}{30}2\left(N_{\rm DC}^2-1\right)T'^4_{\rm conf} = \frac{8}{15}\pi^2 T'^4_0 = \frac{8}{15}\pi^2\left(\frac{g_{\rm today}}{g_{kd}}\right)^{\frac{4}{3}} T^4_{\rm CMB}$$

$$\Omega'_{\rm Dgluons,0}h^2 \approx \frac{1}{2}\Omega'_{\rm CMB,0}h^2 \tag{5.25}$$

5.3.3 Constraints on cosmologically stable glueballs

We have already discussed in section 5.1.3 what are the constraints on glueballs decaying during the BBN or after, with lifetime $1\,\text{s} < \tau_\Phi < 10^{17}\,\text{s}$. We want now to analyse the case of cosmologically stable glueballs and understand if they can be a component of dark matter, accounting for a part of the dark matter density together with the gluequarks . Let us outline the main cosmological bounds.

Upper bound on the mass of stable glueballs from the relic abundance

First of all, the relic abundance of glueballs is bounded from above by the value of the dark matter density parameter, in order not to overclose the Universe.

The density parameter for dark matter measured by Planck [103] is

$$\Omega_{\text{DM}} h^2 = 0.1186 \pm 0.0020$$

This translate into an upper bound on the mass of stable glueballs:

$$M_\Phi \lesssim 0.4\,\frac{M_\Phi}{T'_d}\,\xi\,\text{eV} \tag{5.26}$$

As discussed previously (see equation 5.8), we have the estimates $1 \lesssim \xi \lesssim 4$ and $\frac{M_\Phi}{T'_d} \sim \mathcal{O}(20-40)$. The upper bound on the glueball mass then becomes

$$M_\Phi \lesssim \mathcal{O}(8-64)\,\text{eV}$$

Number of relativistic degrees of freedom during BBN

The upper bound on the mass of stable glueballs analysed in the previous section implies that in the allowed parameter space region with stable glueballs the confinement temperature is below the Big Bang Nucleosynthesis temperature $T_{\text{BBN}} \sim 1\,\text{MeV}$.

At the epoch of BBN the dark sector is then still in the deconfined phase and the dark gluons contribute to the relativistic degrees of freedom, behaving as dark radiation. Extra radiation at the time of BBN modifies the primordial Helium fraction[5] and is constrained by observations.

Usually this effect is parametrised as a contribution to the effective number of neutrinos

$$\Delta N_{\text{eff}} = \frac{4}{7} g'_{\text{BBN}} \left(\frac{T'}{T}\right)^4_{\text{BBN}} \tag{5.27}$$

[5] The expansion rate in the early Universe is fixed by the Friedmann equation $H^2 = 4\pi G_N \rho/3$; in the radiation dominated era it depends on the energy density of relativistic degrees of freedom, with no distinction between dark and visible sector. The presence of dark radiation increases the expansion rate, inducing an earlier neutron-proton freeze-out. This leaves an higher fraction of neutrons available for the synthesis of ^4He.

Inverting the definition $\xi = \frac{gT^3}{g'T'^3}$ we can express the temperature ratio as

$$\left(\frac{T'}{T}\right)^4_{\text{BBN}} = \left(\frac{g}{g'}\right)^{\frac{4}{3}}_{\text{BBN}} \frac{1}{\xi^{\frac{4}{3}}}$$

We can then express ξ as a function of ΔN_{eff} and obtain a bound on ξ from the experimental bound on ΔN_{eff}:

$$\xi = \left(\frac{4}{7}\right)^{\frac{3}{4}} \frac{g_{\text{BBN}}}{\left(g'_{\text{BBN}}\right)^{\frac{1}{4}}} \left(\Delta N_{\text{eff}}\right)^{-\frac{3}{4}}$$

Using the values $g_{\text{BBN}} = 10.75$ and $g'_{\text{BBN}} = 2(N_{\text{DC}}^2 - 1) = 16$ we arrive at

$$\xi = 3.5(\Delta N_{\text{eff}})^{-\frac{3}{4}} \tag{5.28}$$

Using the bound $\Delta N_{\text{eff}} < 1$ at 95% of confidence level from reference [104], we obtain the constraint

$$\xi > 3.5$$

This is only marginally consistent with the estimate for the ξ parameter (equation 5.8) that we obtained in our model $1 \lesssim \xi \lesssim 4$.

Stronger bounds can be found in the literature [105–107] giving $\Delta N_{\text{eff}} < 0.3 \Longrightarrow \xi > 8.5$. This would completely rule out the whole parameter space region with $T'_{\text{conf}} < T'_{\text{BBN}}$, however these bounds have been disputed and so we use the more conservative result of reference [104].

Cosmic Microwave Background radiation constraint

Dark radiation at the time of the last scattering is strongly constrained by Planck 2015 data on the CMB radiation.

These constraints are relevant for the parameter space region in which $T'_{\text{conf}} < T'_{\text{LS}}$, where T'_{LS} is the temperature of the dark sector at the time of the last scattering and is given by $T'_{\text{LS}} = \left(\frac{g_{\text{CMB}}}{g_{kd}}\right)^{\frac{1}{3}} T_{\text{LS}} \approx \frac{1}{2} T_{\text{LS}} \approx 0.1\,\text{eV}$. Since $M_\Phi \sim 7\Lambda_{\text{DC}}$ and $T'_{\text{conf}} \sim \Lambda_{\text{DC}}$, this constraint is relevant for glueballs with mass $M_\Phi \lesssim \mathcal{O}(1)\,\text{eV}$.

Usually this is expressed as a bound on the effective number of neutrinos and the most recent bound at $2\,\sigma$ from Planck [103] is

$$\Delta N_{\text{eff}} < 0.6 \tag{5.29}$$

This translates through equation 5.28 into the bound $\xi > 5$, that is incompatible with the range for ξ of equation 5.8: $1 \lesssim \xi \lesssim 4$. This rules out the whole parameter space region in which $M_\Phi \lesssim \mathcal{O}(1)\,\text{eV}$. However, as we stressed previously, the value of ξ has a large uncertainty, and the observation could still be marginally consistent if $T_{kd} \gtrsim \mathcal{O}(1)\,\text{GeV}$.

Structure formation

There is still a small region in the parameter space of stable glueballs that is not excluded. Using the looser bound from equation 5.26 this corresponds to

$$1\,\mathrm{eV} \lesssim M_\Phi \lesssim 65\,\mathrm{eV}$$

If the glueballs are the main component of dark matter there is a bound on their mass deriving from galaxy formation dynamics: the dark matter thermal velocity at the epoch of galaxy formation $z_{\mathrm{GF}} \sim 5$ should be the typical velocity measured in the galaxies $\beta = \frac{v}{c} \sim 10^{-3}$ [98]. From the virial theorem we have, thus, the condition

$$\frac{3}{2} T'_{\mathrm{GF}} \lesssim \frac{1}{2} M_\Phi \beta^2$$

In order for this condition to be satisfied, galaxy formation must happen after chemical decoupling of the glueball (since $\beta^2 \ll T'_d/M_\Phi$). We can relate the dark sector temperature at galaxy formation to the photon temperature and to the decoupling temperature using the scaling relations: $T'_{\mathrm{GF}}/T'_d \sim a^{-2}$ and $T_{\mathrm{GF}}/T_d \sim a^{-1}$, valid for the glueballs and the photons respectively after chemical decoupling in the glueball sector. We obtain:

$$T'_{\mathrm{GF}} = T'_d \left(\frac{T_{\mathrm{GF}}}{T_d} \right)^2 = T'_d \left(\frac{T_{\mathrm{CMB}}}{T_d} \right)^2 (1 + z_{\mathrm{GF}})^2$$

Combining the two equations we derive the conservative bound:

$$T_d \gtrsim 2\,\mathrm{eV} \left(\frac{T'_d}{M_\Phi} \right)^{\frac{1}{2}} \Rightarrow T_d \gtrsim 0.3\,\mathrm{eV}$$

We need to relate this temperature to the temperature of the dark sector. The calculation is analogous to the one shown in equation 5.22, with T_d in place of T_{CMB}. We obtain the limit on the glueballs mass:

$$M_\Phi \gtrsim 5\,\mathrm{keV}$$

We conclude that glueballs in our model cannot be the main component of dark matter.

There could still be the possibility for the glueballs to be a minority component, together with gluequark $\mathcal{X}$ as main candidate for dark matter. For this to be the case, equation 5.26 gives a decoupling temperature $T_d \lesssim 0.2\,\mathrm{eV}$, or equivalently $\Lambda_{\mathrm{DC}} \lesssim 1\,\mathrm{eV}$. This case is constrained by observations of the matter power spectrum: during the matter dominated era, density perturbations grow forming structures such as galaxies and galaxy clusters; if confinement occurs during or after the epoch of matter radiation equality $T_{\mathrm{eq}} \sim 1\,\mathrm{eV}$, dark gluons form a relativistic fluid that interacts with dark quarks, modifying the matter power spectrum. This possibility has been explored by Schmaltz [108–110] and results in a bound $g_{\mathrm{DC}} \lesssim 10^{-3}$ for typical $M_Q \sim \mathcal{O}(1)$ TeV, corresponding to $\Lambda_{\mathrm{DC}} \lll T_{\mathrm{CMB}}$. This last possibility is excluded in our models by CMB observations on the number of effective neutrinos, with the caveats discussed previously (deriving from the large uncertainty on the kinetic decoupling temperature).

Summary and outlook

From the discussion in the previous paragraphs, we can conclude that in our models the region of the parameter space with stable glueballs is completely excluded by one of the following three reasons:

- the relic density is too large;
- there is too much dark radiation at the epoch of Big Bang Nucleosynthesis or last scattering (CMB);
- structure formation is not compatible with light glueballs or dark matter interacting with dark radiation with a coupling greater than $g_{\rm DC} \sim 10^{-3}$.

Glueball dark matter could be viable if heavier than few keV. In particular, if it is heavier than 1 MeV it can evade also the bound from BBN.

In general, if the dark sector and the standard model sector have been in equilibrium, the ξ parameter will be given by $\xi = \frac{g_{kd}}{g'_{kd}}$. The number of relativistic degrees of freedom in the standard model is 106.75; therefore, if kinetic decoupling happens before confinement in the dark sector, the maximum value for ξ is

$$\xi_{max} = \frac{106.75}{2(N_{\rm DC}^2 - 1)} = 6.7$$

This is not enough to evade the bounds of reference [108–110] and to provide a viable model of dark matter with stable relic glueballs.

A different scenario can be one in which the two sectors have never been in thermal contact with each other. This is the assumption usually made in most of the studies on glueball dark matter (see for instance [111, 112]), in order to accommodate higher glueballs masses $\xi \gtrsim 10^3, \; M_\Phi \gtrsim 1\,\mathrm{MeV}$.

Another possibility is that kinetic decoupling occurs when confinement has already taken place in the dark sector, as put forward on general grounds in [101, 102]. The glueball entropy is suppressed by a Boltzmann factor and the ratio ξ can become large in a parametric way. In our model we could have an explicit realisation of this scenario if there were a parameter space region in which glueballs are stable and heavier than 1 MeV and kinetic decoupling happens later than confinement. This could be the case if there can be stable glueballs with $M_\Phi \gtrsim \mathcal{O}(10)\,\mathrm{MeV}$ and $M_\chi^\pm - M_\chi^0 < M_\Phi$ (see the discussion in section 5.2).

5.4 Gluequark relic density

The thermal relic density of a species is determined by the moment at which the chemical decoupling happens, that is when the interaction rate of number changing processes becomes smaller than the Hubble expansion rate. This event is referred as *freeze-out*: after this moment the number of particles per comoving volume is fixed and the their momentum and number density are just red-shifted by the expansion of the universe.

The freeze-out is determined by the condition $\Gamma \sim H$

$$n_{\text{f.o.}} \langle \sigma v \rangle \sim \frac{T_{\text{f.o.}}^2}{M_{\text{Pl}}} \tag{5.30}$$

A naive estimate of the relic density can be found[6] using the previous equation:

$$\Omega_\chi = \frac{M_\chi n_0}{\rho_{\text{crit}}} = \frac{M_\chi n_{\text{f.o.}}}{\rho_{\text{crit}}} \left(\frac{T_{\text{CMB}}}{T_{\text{f.o.}}} \right)^3 = \frac{T_{\text{CMB}}^3}{\rho_{\text{crit}}} x_{\text{f.o.}} \left(\frac{n_{\text{f.o.}}}{T_{\text{f.o.}}^2} \right) = \left(\frac{T_{\text{CMB}}^3}{\rho_{\text{crit}}\, M_{\text{Pl}}} \right) \frac{x_{\text{f.o.}}}{\langle \sigma v \rangle}$$

where $x_{\text{f.o.}} = M_\chi / T_{\text{f.o.}}$. Inserting the numerical values we arrive at

$$\left(\frac{\Omega_\chi}{0.26} \right) \approx \frac{x_{\text{f.o.}}}{26} \left(\frac{10^{-8}\,\text{GeV}^{-2}}{\langle \sigma v \rangle} \right) \tag{5.31}$$

Using the expression for the number density of a non-relativistic particle in thermal equilibrium

$$n = g \left(\frac{M_\chi T'}{2\pi} \right)^{\frac{3}{2}} e^{-\frac{M_\chi}{T'}} \sim \frac{M_\chi^3}{x^{3/2}} e^{-x}$$

the value of $x_{\text{f.o.}}$ can be obtained by making again use of equation 5.30:

$$\begin{aligned} \frac{M_\chi^3}{x^{3/2}} e^{-x} &\sim \frac{M_\chi^2}{x^2 M_{\text{Pl}} \langle \sigma v \rangle} \\ \sqrt{x} e^{-x} &\sim \frac{1}{M_{\text{Pl}} M_\chi \langle \sigma v \rangle} \end{aligned}$$

Solving numerically this equation for typical values of $\langle \sigma v \rangle$ and M_χ one finds that

$$x_{\text{f.o.}} \sim \mathcal{O}(20 - 40) \tag{5.32}$$

with a weak logarithmic dependence on $M_\chi \langle \sigma v \rangle$.

Therefore the dark matter relic abundance $\Omega_{\text{DM}} \sim 0.26$ is reproduced by equation 5.31 if the annihilation cross section is of order

$$\langle \sigma v \rangle \sim 10^{-8}\,\text{GeV}^{-2}$$

For a more careful calculation one has to solve the Boltzmann equation [113]. For the annihilation of two identical particles this reads

$$\frac{\mathrm{d}n}{\mathrm{d}t} + 3Hn = -\langle \sigma v \rangle \left(n^2 - n_{\text{eq}}^2 \right)$$

where n is the number density of dark matter particles, n_{eq} its equilibrium value and $\langle \sigma v \rangle$ the

[6] In this calculation we are assuming that the dark matter candidate is non-relativistic at freeze-out, *i.e.* it is a *cold* relic.

thermally averaged annihilation cross section defined as

$$\langle \sigma v \rangle = \frac{\int \sigma v \, \mathrm{d}n_1^{\mathrm{eq}} \mathrm{d}n_2^{\mathrm{eq}}}{\int \mathrm{d}n_1^{\mathrm{eq}} \mathrm{d}n_2^{\mathrm{eq}}}$$

It is customary to reabsorb the term due to the Hubble expansion $3Hn$ using the variable $Y = n/s$, where s is the comoving entropy and entropy conservation dictates $\frac{\mathrm{d}s}{\mathrm{d}t} + 3Hs = 0$. Using the scale factor a as a time coordinate and the definition of Hubble parameter $H = \dot{a}/a$ we arrive at

$$\frac{\mathrm{d}Y}{\mathrm{d}a} = -\frac{s \langle \sigma v \rangle}{aH} \left(Y^2 - Y_{\mathrm{eq}}^2 \right)$$

As a last step we rewrite the Boltzmann equation as a function of the variable $x = M_\chi/T$, where T is the photon temperature[7] obtaining:

$$\frac{\mathrm{d}Y}{\mathrm{d}x} = -\frac{s \langle \sigma v \rangle}{Hx} \left(Y^2 - Y_{\mathrm{eq}}^2 \right) \tag{5.33}$$

This equation must be solved numerically or through approximate analytical methods in order to find the dark matter relic abundance.

5.4.1 Annihilation cross section

The annihilation of χ can take place through two different channels: dark gluons (for all the dark quarks) or Standard Model particles (for the dark quarks charged under $\mathcal{G}_{\mathcal{SM}}$).

The relevant scale for the annihilation is $T_{\mathrm{f.o.}} \sim M_{\mathcal{Q}}/20 \gg \Lambda_{\mathrm{DC}}$ in the allowed parameter space region, therefore we are still in a perturbative regime.

The cross section for the annihilation of two dark quarks in two dark gluons can be estimated as

$$\sigma \sim 4\pi \frac{\alpha_{\mathrm{DC}}^2}{M_\chi^2}$$

while for the annihilation in Standard Model particles one has

$$\sigma \sim 4\pi \frac{1}{N_{\mathrm{DC}}^2} \frac{\alpha_{\mathrm{EW}}^2}{M_\chi^2}$$

The additional factor $1/N_{\mathrm{DC}}^2$ is due to the average over initial states: dark quarks transform as the adjoint of $\mathrm{SU}(N)_{\mathrm{DC}}$ and have thus a multiplicity of $(N_{\mathrm{DC}}^2 - 1)$. Since the final state is a dark colour singlet we get a suppression factor of order $1/N_{\mathrm{DC}}^2$.

In our models, the parameter space region compatible with a fast decay of the glueballs and consistent with observations from BBN and CMB corresponds to $\alpha_{\mathrm{DC}} \sim 0.1$, while the electroweak coupling gives $\alpha_{\mathrm{EW}} \sim 0.03$. Comparing the two estimates, for $N_{\mathrm{DC}} = 3$, we find that the annihilation cross section in dark gluons is two orders of magnitude stronger than the one in Standard Model particles. We thus neglect the latter, focusing our attention on the former.

[7] As we discussed in section 5.2, at freeze-out the dark sector is in thermal equilibrium with the visible sector, therefore we have a single thermal bath.

Calculation of the annihilation cross section in dark gluons

We consider the annihilation of a pair of Weyl adjoint fermions (dark quarks) in two dark gluons. For simplicity we consider the case in which the dark quark is a Standard Model singlet (this corresponds to the field N defined in section 4.3).

There are three diagrams that contribute to the annihilation cross section at tree level: t-channel and u-channel annihilation (with a virtual dark quarks) and s-channel annihilation (with a virtual dark gluon).

Using the lagrangian 4.31, one obtains the following amplitudes for the three diagrams:

$$\begin{aligned}
i\mathcal{M}_1 &= -ig_{\mathrm{DC}}^2 \bar{v}(p_2)\gamma^\nu \frac{\not{p}_1 - \not{k}_1 + M}{t - M^2}\gamma^\mu u(p_1)\varepsilon_{1\mu}^*\varepsilon_{2\nu}^*\left(T_{\mathrm{adj}}^d T_{\mathrm{adj}}^c\right)_{ba} \\
i\mathcal{M}_2 &= -ig_{\mathrm{DC}}^2 \bar{v}(p_2)\gamma^\mu \frac{\not{p}_1 - \not{k}_2 + M}{u - M^2}\gamma^\nu u(p_1)\varepsilon_{1\mu}^*\varepsilon_{2\nu}^*\left(T_{\mathrm{adj}}^c T_{\mathrm{adj}}^d\right)_{ba} \\
i\mathcal{M}_3 &= -ig_{\mathrm{DC}}^2 \bar{v}(p_2)\gamma^\rho u(p_1)\frac{1}{s^2}\left[g^{\mu\nu}(k_2 - k_1)_\rho + g_\rho^\nu(-k_1 - 2k_2)^\mu + g_\rho^\mu(2k_1 + k_2)^\nu\right] \\
&\qquad \varepsilon_{1\mu}^*\varepsilon_{2\nu}^*\left(T_{\mathrm{adj}}^e\right)_{ba}(f_{cde})
\end{aligned} \tag{5.34}$$

where s, t, u are the Mandelstam variables.

Squaring the amplitude we have

$$|\mathcal{M}|^2 = |\mathcal{M}_1|^2 + |\mathcal{M}_2|^2 + |\mathcal{M}_3|^2 + 2\,\mathrm{Re}(\mathcal{M}_1\mathcal{M}_2^*) + 2\,\mathrm{Re}(\mathcal{M}_1\mathcal{M}_3^*) + 2\,\mathrm{Re}(\mathcal{M}_2\mathcal{M}_3^*)$$

Summing over gluon polarisations λ we need to evaluate the sum $\sum_\lambda \varepsilon_{1\mu}^*\varepsilon_{1\rho}$ and similarly for ε_2. The sum is restricted to the physical polarisations of the gluons in the final state, namely transverse modes. Usually in QED one takes advantage of the Ward identity $k_{1\mu}\mathcal{M}^{\mu\nu} = k_{2\nu}\mathcal{M}^{\mu\nu} = 0$ and replace this sum with the sum over transverse *and longitudinal* polarisations

$$\sum_{\substack{\text{transverse,}\\ \text{longitudinal}}} \varepsilon_{1\mu}^*\varepsilon_{1\rho} = -g_{\mu\rho}$$

In words, this is equivalent to say that in QED the Ward identity ensures that the amplitude to produce on-shell longitudinal photons in the final state from an initial state with no longitudinal photons is identically zero[8].

In the non-abelian case more care is needed. The naive Ward identity is no more valid and it must be replaced with Slavnov-Taylor identities [114, 115] such as

$$\alpha_{\mathrm{DC}}^{-1}\langle T\partial_\mu A_\mu^a(x)A_\nu^b(y)\rangle = -i\langle T\bar{c}^a(x)(D_\nu c)^b(y)\rangle$$

where $c^a(x)$ is the Faddeev-Popov ghost. This implies that $k_{1\mu}\mathcal{M}^{\mu\nu} \neq 0$, $k_{2\mu}\mathcal{M}^{\mu\nu} \neq 0$. In particular, the amplitude for producing a couple of unphysical longitudinally polarised gluons is non-zero, while the amplitude to produce a transverse gluon together with a longitudinal gluon is zero as for the usual case (see [67] for a discussion).

[8] This is true at each loop order, but only once all the diagrams have been included (*i.e.* it is not true diagram by diagram).

As a result, we cannot sum over physical and unphysical polarisation because these would give a spurious contribution to the cross section, violating the unitarity of the theory. This was first noted by Feynman in 1963 ([116]), long before QCD was formulated and recognised as the fundamental theory of strong interactions, and led to the introduction of Faddeev-Popov gosths few years later [117].

Three possible routes can be chosen (all yielding the same result[9])

- perform explicitly the sum over transverse polarisations in the reference frame of interest. This is straightforward but computationally cumbersome.
- sum over physical and unphysical polarisation and add the (negative) cross section to produce a ghost-antighost pair, in order to recover unitarity.
- modify the three-gluons vertex in such a way that the amplitude for physical polarisations is unchanged $\mathcal{M}^{\mu\nu}\varepsilon^*_{1\mu}\varepsilon^*_{2\nu} = \widetilde{\mathcal{M}}^{\mu\nu}\varepsilon^*_{1\mu}\varepsilon^*_{2\nu}$ but now, differently from before, $k_{1\mu}\widetilde{\mathcal{M}}^{\mu\nu} = k_{2\nu}\widetilde{\mathcal{M}}^{\mu\nu} = 0$. This was first suggested by Feynman in [116].

We choose to pursue the last route and modify the gluon vertex, following [119]. The amplitude $i\mathcal{M}_3$ becomes:

$$i\widetilde{\mathcal{M}}_3 = -ig^2_{\text{DC}}\bar{v}(p_2)\gamma^\rho u(p_1)\frac{1}{s^2}\left[g^{\mu\nu}(k_2-k_1)^\rho - 2g^{\nu\rho}k_2^\mu + 2g^{\rho\mu}k_1^\nu\right]\varepsilon^*_{1\mu}\varepsilon^*_{2\nu}\left(T^e_{\text{adj}}\right)_{ba}(f_{cde})$$

We can now replace the sum over physical polarisations with a sum over transverse and longitudinal polarisations, and use $\sum\varepsilon^*_\mu\varepsilon_\rho = -g_{\mu\rho}$.

To calculate the cross section we square the amplitude, sum over final state polarisations and colours indices and average over initial state spin and colour indices. The following group theory factors are needed in the calculation:

$$\begin{aligned}
\text{Tr}\left(T^d_{\text{adj}}T^d_{\text{adj}}T^c_{\text{adj}}T^c_{\text{adj}}\right) &= \left(N^2_{\text{DC}}-1\right)N^2_{\text{DC}} \\
\text{Tr}\left(T^d_{\text{adj}}T^c_{\text{adj}}T^d_{\text{adj}}T^c_{\text{adj}}\right) &= \frac{\left(N^2_{\text{DC}}-1\right)N^2_{\text{DC}}}{2} \\
\text{Tr}\left(T^e_{\text{adj}}T^h_{\text{adj}}\right)\text{Tr}\left(T^e_{\text{adj}}T^h_{\text{adj}}\right) &= \left(N^2_{\text{DC}}-1\right)N^2_{\text{DC}} \\
\text{Tr}\left(T^c_{\text{adj}}T^d_{\text{adj}}T^e_{\text{adj}}\right)\left(if^{cde}\right) &= -\frac{\left(N^2_{\text{DC}}-1\right)N^2_{\text{DC}}}{2}
\end{aligned}$$

Using the previous results and performing the Dirac algebra we arrive at the averaged squared

[9] Explicit calculations in the context of QCD were carried to evaluate the cross section for the production of charmed particles from gluon-gluon annihilation in proton-antiproton collisions. The first two methods were used in [118], while the third in [119, 120]. All the results are in agreement.

amplitude[10]

$$\langle|\mathcal{M}|^2\rangle = \frac{1}{4}\frac{1}{(N_{\mathrm{DC}}^2-1)^2}\sum|\mathcal{M}|^2 = 36\,\pi^2\alpha^2\left(\frac{2(M_Q^2-t)(-M_Q^2+s+t)}{s^2}+\frac{M_Q^2(s-4M_Q^2)}{(M_Q^2-t)(-M_Q^2+s+t)}-\right.$$
$$-\frac{M_Q^2(2M_Q^2-s-2t)+(M_Q^2-t)(-M_Q^2+s+t)}{s(M_Q^2-t)}+\frac{(M_Q^2-t)(-M_Q^2+s+t)-2M_Q^2(3M_Q^2-s-t)}{(-M_Q^2+s+t)^2}+$$
$$\left.+\frac{(M_Q^2-t)(-M_Q^2+s+t)-2M_Q^2(M_Q^2+t)}{(M_Q^2-t)^2}-\frac{M_Q^2(-2M_Q^2+s+2t)+(M_Q^2-t)(-M_Q^2+s+t)}{s(-M_Q^2+s+t)}\right)$$

where we used the kinematic identity $s+t+u=2M_Q^2$ to eliminate u.

The differential cross section in the center of mass frame is then given by

$$\frac{\mathrm{d}\sigma(s,t)}{\mathrm{d}t} = \frac{1}{2}\frac{1}{64\pi s}\frac{1}{|p_1|^2}\langle|\mathcal{M}|^2\rangle \tag{5.35}$$

and the total cross section is obtained integrating on t:

$$\sigma(s) = \frac{1}{2}\frac{1}{64\pi s}\frac{1}{|p_1|^2}\int_{T_{\min}}^{t_{\max}}\langle|\mathcal{M}|^2\rangle dt \tag{5.36}$$

where the $1/2$ factor accounts for the presence of two identical particles (dark gluons) in the final state. After some relativistic kinematics one finds

$$|p_1|^2 = \frac{s}{4}\left(1-\frac{4M_Q^2}{s}\right)$$
$$t_{\max} = M_Q^2-\frac{s}{2}\left(1-\sqrt{1-\frac{4M_Q^2}{s}}\right)$$
$$t_{\min} = M_Q^2-\frac{s}{2}\left(1+\sqrt{1-\frac{4M_Q^2}{s}}\right)$$

Defining $\Delta=\sqrt{1-4M_Q^2/s}$ and evaluating the integral, the cross section can be expressed as

$$\sigma(s) = \frac{3\pi}{4}\frac{\alpha_{\mathrm{DC}}^2}{s}\frac{1}{\Delta^2}\left[3\left(1+\frac{4M_Q^2}{s}-\frac{4M_Q^4}{s^2}\right)\log\frac{1+\Delta}{1-\Delta}-\left(4+\frac{17M_Q^2}{s}\right)\Delta\right] \tag{5.37}$$

This result is consistent with a similar calculation carried out in the context of MSSM, the cross section for the production of a pair of gluinos from the annihilation of two gluons, once the two cross sections are related through crossing symmetry [121, 122].

We are interested in the non relativistic limit of this cross section. Therefore we perform a

[10] We evaluate the colour factors for $N_{\mathrm{DC}}=3$, the relevant case for the models we are studying.

Laurent expansion in powers of the Møller velocity

$$v = \frac{\sqrt{(p_1 \cdot p_2)^2 - (M_1 M_2)^2}}{E_1\, E_2} = 2\sqrt{1 - \frac{4M_{\mathcal{Q}}^2}{s}} = 2\Delta \;\Rightarrow\; s = \frac{4M_{\mathcal{Q}}^2}{1-\frac{v^2}{4}}$$

where we specialised to the center of mass frame and used some relativistic kinematics. It follows that

$$\sigma = \frac{27\pi}{32}\frac{\alpha_{\rm DC}^2}{M_{\mathcal{Q}}^2}\frac{1}{v} + \frac{15\pi}{64}\frac{\alpha_{\rm DC}^2}{M_{\mathcal{Q}}^2}v - \frac{309\pi}{2560}\frac{\alpha_{\rm DC}^2}{M_{\mathcal{Q}}^2}v^3 + \mathcal{O}\left(v^5\right) \tag{5.38}$$

The first term of the expansion (usually referred as the s-wave annihilation cross section) agrees with the one quoted in [123].

We then calculate the thermal average following [113] and obtain our final result:

$$\langle \sigma v\rangle[x] = \frac{27\,\pi}{32}\frac{\alpha_{\rm DC}^2}{M_{\mathcal{Q}}^2} + \frac{9\,\pi}{64}\frac{\alpha_{\rm DC}^2}{M_{\mathcal{Q}}^2}\frac{1}{x} - \frac{963\,\pi}{128}\frac{\alpha_{\rm DC}^2}{M_{\mathcal{Q}}^2}\frac{1}{x^2} + \mathcal{O}\left(x^{-3}\right) \tag{5.39}$$

where $x = M_{\mathcal{Q}}/T \simeq M_\chi/T$.

We define, for later convenience, σ_0 as the thermally averaged *s-wave* annihilation cross section, corresponding to the first term of the previous expansion:

$$\sigma_0 = \frac{27\,\pi}{32}\frac{\alpha_{\rm DC}^2}{M_{\mathcal{Q}}^2} \tag{5.40}$$

5.4.2 Sommerfeld enhancement

The tree level annihilation cross section computed in the last paragraph can have large non-perturbative corrections due to the so called *Sommerfeld enhancement* [124, 125]. Long range interactions among the incoming particles can strongly distort their wave function, yielding significant increase (if the long range interaction is attractive) or decrease (if it is repulsive) of their annihilation cross section.

For s-wave annihilation, in the non relativistic limit the cross section is proportional to the modulus squared of the reduced two body wave function at the origin

$$\sigma \propto |\psi_k(0)|^2$$

The Sommerfeld enhancement factor with respect to the tree level cross section can then be expressed as

$$S_k = \frac{|\psi_k(0)|^2}{|\psi_k^{(0)}(0)|^2} = |\psi_k(0)|^2$$

where $\psi_k^{(0)}(\vec{x}) = e^{i\vec{k}\cdot\vec{x}}$ is the plane-wave approximation for the unperturbed case while $\psi_k(\vec{x})$ takes into account long range interactions among the two incoming particles and can be determined by solving the Schrödinger equation.

For an abelian U(1) gauge group with massless mediator, generating Coulomb potential

$$V(r) = \frac{\alpha}{r}$$

the Sommerfeld correction [124, 125] is given by

$$S_{\text{abelian}}\left(\frac{\alpha_{\text{DC}}}{\beta}\right) = \frac{-\pi\alpha_{\text{DC}}/\beta}{1 - e^{-\pi\alpha_{\text{DC}}/\beta}}$$

where β is the velocity of an incoming particle.

The non-abelian case can be reduced to the abelian one decomposing the tensor product of the two representations of the incoming particles (R_1 and R_2) into a sum of irreducible representations [126]. For each irreducible representation R'_i the potential is given by:

$$V(r) = \frac{\alpha_{\text{DC}}}{2r}\left(\sum_i C_2(R'_i) - C_2(R_1) - C_2(R_2)\right)$$

Using the result of reference [123, 127] for the decomposition of the product of two adjoints representations one has the following Sommerfeld enhancement factor for the annihilation cross section of two dark quarks:

$$S_{\text{ann}} = \frac{1}{6}S_{\text{abelian}}\left(-3\frac{\alpha_{\text{DC}}}{\beta}\right) + \frac{1}{3}S_{\text{abelian}}\left(-\frac{3}{2}\frac{\alpha_{\text{DC}}}{\beta}\right) + \frac{1}{2}S_{\text{abelian}}\left(\frac{\alpha_{\text{DC}}}{\beta}\right) \tag{5.41}$$

5.4.3 Relic density and mass of the candidate

To compute the dark matter relic density we need to solve the Boltzmann equation 5.33

$$\frac{\mathrm{d}Y}{\mathrm{d}x} = -\frac{s\langle\sigma v\rangle}{Hx}\left(Y^2 - Y_{\text{eq}}^2\right)$$

We follow reference [127] and use an approximate analytical solution for the asymptotic relic density.

We can rewrite the Boltzmann equation as

$$\frac{\mathrm{d}Y}{\mathrm{d}x} = -\frac{s\,\sigma_0}{H}\frac{\langle\sigma v\rangle}{\sigma_0}\frac{1}{x}\left(Y^2 - Y_{\text{eq}}^2\right)$$

where σ_0 is the thermally averaged s-wave annihilation cross section computed at tree-level, defined in equation 5.40. During the radiation dominated epoch we have

$$\frac{s\,\sigma_0}{H} \propto T \Longrightarrow \frac{s\,\sigma_0}{H} = \left.\frac{s\,\sigma_0}{H}\right|_{T=M_\chi}\frac{T}{M_\chi} = \lambda\frac{1}{x}$$

Using the Friedmann equation and expressing the entropy density we can write λ as

$$\lambda = \sigma_0\left(\frac{2\pi^2}{45}g_{\text{TOT}}M_\chi^3\right)\sqrt{\frac{3M_{\text{Pl}}^2}{8\pi}\frac{30}{\pi^2 g_{\text{TOT}}}\frac{1}{M_\chi^4}} = \sigma_0\sqrt{\frac{g_{\text{TOT}}\pi}{45}}M_{\text{Pl}}M_\chi$$

where $g_{\text{TOT}} = g_{\text{SM}} + g_{\text{DC}} = 106.75 + 16 = 122.75$ is the effective number of relativistic degrees of freedom at temperature $T \sim M_Q$.

The factor $\langle \sigma v \rangle / \sigma_0$ corresponds to the Boltzmann enhancement $S(x)$ discussed previously. Therefore we obtain our final form for the Boltzmann equation

$$\frac{\mathrm{d}Y}{\mathrm{d}x} = -\frac{\lambda S(x)}{x^2} \left(Y^2 - Y_{eq}^2 \right) \tag{5.42}$$

An approximate asymptotic solution has been obtained in [127]:

$$Y(\infty) = \frac{1}{\lambda} \left(\int_{x_f}^{\infty} \frac{S(x)}{x^2} \mathrm{d}x + \frac{S(x_f)}{x_f^2} \right)^{-1} \tag{5.43}$$

where x_f is defined by the relation

$$x_f = \ln \left(\frac{2 g_\chi S(x_f) \lambda}{(2\pi x_f)^{3/2}} \right)$$

and can be identified with the scale at which freeze-out occurs[11].

The relic abundance can then be expressed as:

$$\Omega_{\text{DM}} = \frac{\rho_{\text{DM}}}{\rho_{\text{crit}}} = \frac{n_{\text{DM}} M_\chi}{3 H_0^2 M_{\text{Pl}}^2 / 8\pi} = \frac{Y(\infty) s_0 M_\chi}{3 H_0^2 M_{\text{Pl}}^2 / 8\pi}$$

where we have used the definition of $Y = n_{\text{DM}} / s_{\text{TOT}}$ and the fact that the present day entropy is dominated by the cosmic background radiation photons (there is no dark radiation, since the gluons confine in glueballs and then decay quickly), so that $s_{\text{TOT}} = s_0$.

Inverting the previous relation, using equation 5.43 and the definition of λ we obtain an expression for M_χ:

$$M_\chi^2 = \frac{\Omega_{\text{DM}} h^2}{0.110} \sqrt{\frac{g_{\text{TOT}} \pi}{45}} \left(\sigma_0 M_\chi^2 \right) \left(\int_{x_f}^{\infty} \frac{S(x)}{x^2} \mathrm{d}x + \frac{S(x_f)}{x_f^2} \right) \left(4 \times 10^3 \, \text{TeV}^2 \right) \tag{5.44}$$

For a dark quark singlet under the Standard Model (*i.e.* N), the thermally averaged s-wave cross is given by equation 5.40. The Q^2 of this process is of order M_Q, so we use the value of the dark colour coupling constant at the scale M_Q:

$$\sigma_0 = \frac{27\pi}{32} \frac{1}{M_Q^2} \left(\frac{2\pi}{11 \log\left(\frac{M_Q}{\Lambda_{\text{DC}}} \right)} \right)^2$$

If the dark quark is an $\text{SU}(2)_{\text{EW}}$ multiplet of dimension L, the cross section is modified by an additional factor $1/L$ (due to the average over initial state), giving a relic density enhanced by a factor of L, if M_Q is fixed.

[11] We should say, more properly, that this is the scale at which the qualitative behaviour of the solution changes, since the freeze-out is not instantaneous.

The Sommerfeld enhancement is given by equation 5.41. In this case, since the typical transferred Q^2 is of order $\alpha_{DC} M_Q$ we use the dark colour coupling renormalized at this scale. Moreover, using the virial theorem, we express the velocity β as a function of $x = M_Q/T$: $\beta \simeq \sqrt{3/x}$.

At a certain temperature T_{conf} confinement occurs. We cut-off the integral at $x_{\text{conf}} \sim M_Q/\Lambda_{\text{DC}}$, where we have estimated the confinement temperature to be of order Λ_{DC}. In our model the dark quarks transform as the adjoint representation, therefore they can form a bound state with a gluon. We assume that nearly all the dark quarks confine in gluequarks, which have a mass $M_\chi \simeq M_Q$, and that the recombination in di-quarks or multi-quarks is negligible. We assume, moreover, that after confinement the annihilation of gluequarks gives negligible effects[12].

Our calculation is valid in the hypothesis that confinement occurs after freeze-out. If this is not the case, a more careful analysis is needed. The freeze-out temperature can be estimated solving the previous relation and in our model we obtain $x_f \approx 27$. We consider our calculation valid if $\Lambda_{\text{DC}} < T_f/4$, *i.e.* $x_{\text{conf}} \gtrsim 100$.

Solving iteratively equation 5.44 we can determine the value of the dark matter candidate that reproduces the observed relic density as a function of Λ_{DC}.

VNN model

In this model all the neutral gluequarks V^0, N_1, N_2 are separately accidentally stable due to the three accidental $\mathbb{Z}_2$ symmetries. Non-renormalizable operators (of dimension 6) are expected to induce their decay with a lifetime greater than the age of the universe, as discussed in section 4.3. The charged components of the triplet are heavier than the neutral one by $\Delta M \approx 150\,\text{MeV}$ [83] and decay to the neutral component through weak interactions after the temperature drops below 150 MeV, with a short lifetime.

The dark matter is composed by the three neutral gluequarks; if the three gluequarks have similar masses, since the relic density is determined just by dark colour interactions, the three species will be present with the ratio $V^0 : N_1 : N_2 = 3 : 1 : 1$. Since V^0 is the neutral component of an electroweak triplet it has electroweak interactions.

LLN model

As we discussed in section 4.3.2, in this model the only neutral accidentally stable gluequark is N_3, under the conditions that $M_L \geq M_N$ and $y \leq 10^{-5}$.

The charged component of the doublets L_1 and L_2 decays dominantly into the neutral mass eigenstates N_1, N_2 through weak interactions. These in turn decay to N_3 through flavour changing neutral currents, mediated by the Yukawa interactions in the dark sector, as discussed in section.

The relic density is given by N_3 which is the mass eigenstate predominantly composed by the Standard Model singlet dark quark N, but has non-zero coupling to the Higgs and Z bosons due to the mixing with the doublets (see section 4.3.2.).

[12]The annihilation cross section for the confined states could be enhanced [35]. A dedicated study of this effect would be needed in order to understand if it is relevant or not in this model.

Results and discussion

The results of our calculation of the dark matter candidate mass are shown in figure 5.1a. In the graph we display also the following regions and boundaries:

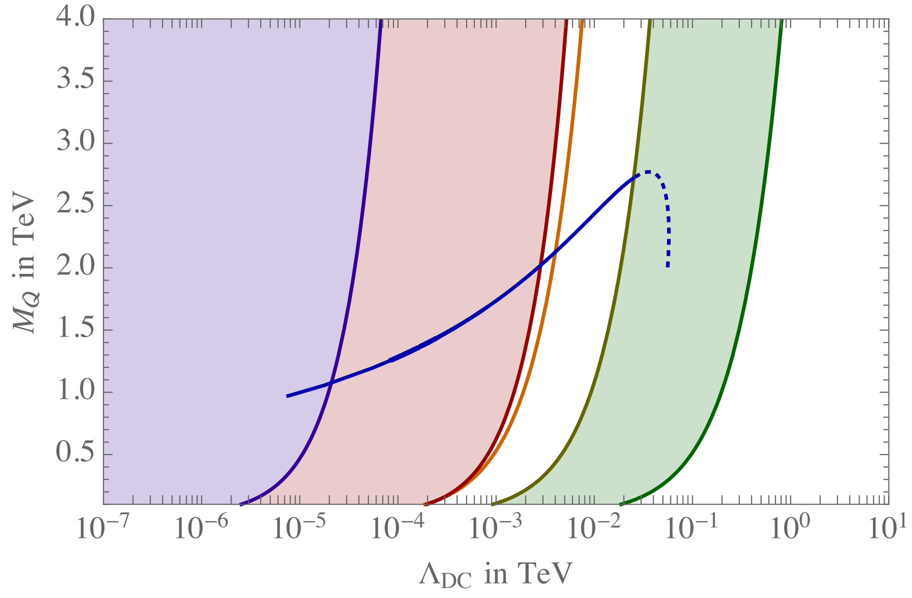
- green curve corresponding to the upper bound on $\Lambda_{\rm DC}$, coming from the requirement that the Landau pole of the dark colour gauge coupling, if present, occurs at a scale higher than the Planck mass (as detailed is section 4.1): $\Lambda_{\rm DC} < M_Q \exp\Big(-\sqrt{b_1 \log(M_Q/M_{\rm Pl})}/(4\, b_0|_{\rm YM})\Big)$;
- green-shaded area: in this region freeze-out and confinement occur approximately at the same epoch and our calculation is not reliable. The left boundary is given by: $\Lambda_{\rm DC} < T_f/4$;
- orange curve: gauge coupling constant at the scale M_Q corresponding to the value of the would-be conformal infrared fixed point $\Lambda_{\rm DC} = M_Q \exp(b_1/(2b_0\; b_0|_{\rm YM}))$;
- red-shaded area: region in which the glueballs decay during or after BBN and are unconstrained by cosmological or astrophysical observations (see section 5.1). The right boundary corresponds to the condition: $\tau_\Phi = 1\,{\rm s}$;
- blue-shaded area: region in which the glueballs are cosmologically stable. This region is excluded due to cosmological observations, as discussed is section 5.3. The right boundary corresponds to the condition $\tau_\Phi = 10^{17}\,{\rm s}$.

For the model LLN, in the case in which the Yukawa coupling is small $y \leq 10^{-5}$ (so that the gluequark N_3 is cosmologically stable), the glueball decay width, computed in section 5.1, is dominated by the decay induced by the operator $\mathcal{O}_8$. Therefore, the glueball lifetime boundaries are unmodified for the two models, in the relevant parameter space region and figure 5.1a is representative for both. For comparison, we show also the corresponding graph if Yukawa couplings of order one were allowed (figure 5.1a). For instance, this possibility could be realised forbidding the dimension 5 non-renormalizable operator responsible for the gluequarks (discussed in section 4.3.2) by imposing the $\mathbb{Z}_2^{\rm DC}$ symmetry.

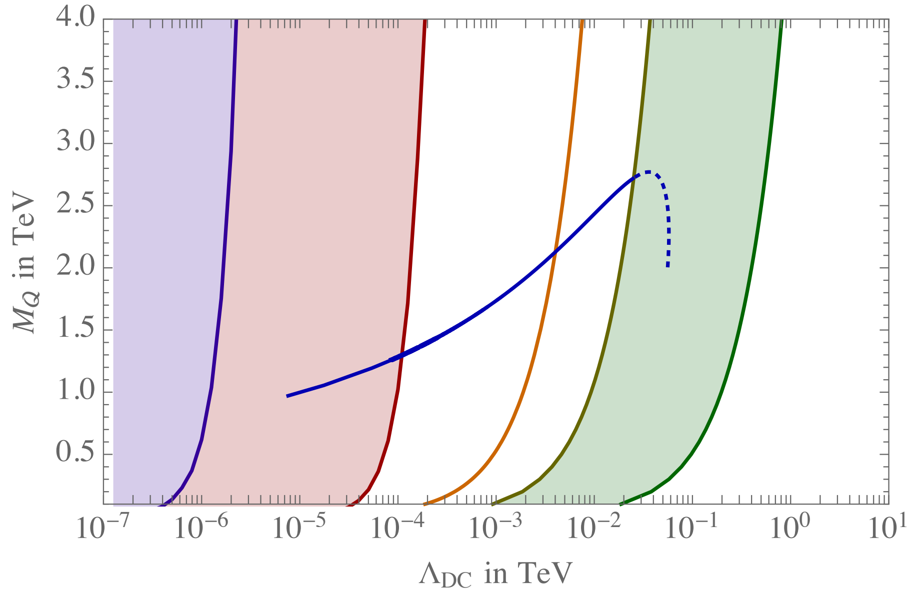
In the graph we show the mass M_Q that the candidate should have in order to reproduce the correct thermal abundance, as a function of $\Lambda_{\rm DC}$. We note that in the region in which freeze-out and confinement overlap, the mass of the candidate drops, due to the fact that the annihilation lasts for a shorter time. Indeed in this region even though the coupling increases, the annihilation is less efficient; as a result the relic number density of dark matter particles is larger, requiring a smaller mass. This can be seen from equation 5.43: there are two competing effects, the rise of the cross section σ_0 due to the higher coupling and the fall of the factor in parenthesis.

In the region in which our calculations are valid, we find viable dark matter candidates if the dark quarks have a mass in the range $M_Q = (2 \div 3)\,{\rm TeV}$, with a confinement scale $\Lambda_{\rm DC} = (10 \div 100)\,{\rm GeV}$. We have not included non-perturbative corrections due to bound-state formation which could lift the mass of the candidate.

From the graph we see that cosmological and astrophysical bounds on long lived decaying relics such as glueballs [93, 94] exclude almost all the parameter space region corresponding to



(a) Parameter space region, dark matter candidate mass and cosmological bounds for the model VNN, and the model LLN with Yukawa coupling $y < 10^{-5}$, for which there is a cosmologically accidentally stable gluequark.



(b) Parameter space region, dark matter candidate mass and cosmological bounds for the model LLN with Yukawa coupling $y \sim 1$, for which the cosmological stability of the *gluequark* can be obtained imposing a $\mathbb{Z}_2^{\rm DC}$ symmetry. This graph is presented for comparison with the previous one.

Figure 5.1: Value of the dark quark mass M_Q necessary to reproduce the correct thermal relic abundance as a function of the confinement scale $\Lambda_{\rm DC}$. Cosmological bounds on the parameter space M_Q versus $\Lambda_{\rm DC}$ are represented: the blue shaded region corresponds to the regime with stable glueballs, which is excluded by BBN or relic abundance; the red region corresponds to the regime with long-lived glueballs decaying during or after BBN, which is excluded by astrophysical observations. The orange curve corresponds to the condition $g(M_Q) = g_* \approx 1.07$, *i.e.* the approximate infrared fixed point of the model. The green shaded region can be viable, but the calculation of the dark matter relic density is no more reliable since confinement occurs at the same epoch as freeze-out. Finally, the upper bound on $\Lambda_{\rm DC}$ corresponds to the condition that the dark colour gauge coupling does not have a Landau pole below $M_{\rm Pl}$.

the first branch of the β function (*i.e.* in which the dark colour coupling at the mass scale $M_{\mathcal{Q}}$ is smaller than g_* and the model is asymptotically free).

The dark matter candidate in this model falls naturally in the TeV, while the region with dark quarks with masses heavier than a few TeV is excluded. We stress that the candidate is not a standard WIMP, since its relic abundance is not determined by electroweak interactions but by dark colour interactions. Electroweak interactions play, however, a fundamental role in keeping the two sector in kinetic equilibrium; this is a crucial point that sets the relevant mass scale at the TeV for annihilation proceeding through interactions with coupling of order $\alpha \sim 1/10$.

The mass range of interest is much lower than the one typical of composite dark matter models realising vectorlike confinement [1], which falls in the 100 TeV range saturating the perturbative unitarity bound for the mass of thermal relics [128]. This is due to the fact that in our model the confinement occurs when the dynamics is still perturbative, while in the other case the annihilation of dark baryons takes place in a non-perturbative regime, with large *hadron-like* annihilation cross sections.

Moreover, the candidates of our model are lighter also with respect to the baryonic dark matter candidates of models with heavy dark quarks transforming as the fundamental representation of the dark colour gauge group [35], which are typically in the mass range of $(10 \div 100)$TeV. This mismatch can be traced back to the following reasons:

- gluequarks have a mass $M_\chi \sim M_{\mathcal{Q}}$ while baryons with heavy dark quarks have a mass $M_{\mathcal{B}} \sim N_{\rm DC} M_{\mathcal{Q}}$;
- the tree-level annihilation cross section and the Sommerfeld enhancement factor are modified by group theory factors due to the difference in the representations of the dark quarks;
- at the confinement transition, adjoint dark quarks confine predominantly in gluequarks while dark quarks in the fundamental representation can confine in (cosmologically unstable) mesons and baryons with comparable probabilities;
- our calculations do not include corrections to the annihilation cross section due to bound state formation [127], nor the effect of annihilation after confinement.

The mass range we have obtained is interesting also from the phenomenological point of view, since it lies at the boundary of the current collider and direct detection sensitivities. Further studies will be needed to address the collider phenomenology and the direct detection signatures.

Conclusions

In this work we have studied theories with a dark sector featuring a non-Abelian gauge dynamics that induces accidentally stable composite dark matter candidates. We have identified two classes of theories, namely chiral models and models with infrared fixed points, that have received little attention in the literature and that can provide new scenarios for composite dark matter, with a dynamics much different from the one realised in theories with vectorlike confinement.

This work represents an introductory study for the dynamics of the two classes of models that we have identified. After discussing some general properties of chiral models, we have shown that under quite general assumptions, in the absence of fundamental scalar fields, models based on $\mathrm{SU}(N)_{\mathrm{DC}}$ gauge group have massless or light states, suggesting a few mechanisms to lift their masses.

In the second part of the thesis we have considered models with an infrared fixed point. We have identified two models with a simple field content (five adjoint Weyl fermions) which have a perturbative fixed point and we have studied their dynamics. The explicit mass term of the dark quarks drives the theory away from the fixed point, breaking the approximate conformal invariance and leaving a confining dynamics in the infrared. If the dark quarks have all the same mass, the dynamics of the model automatically gives a confining gauge theory with dark quarks heavier than the confinement scale. Below the confinement scale, the relevant asymptotic states are given by gluequarks (dark gluon-quark bound states) and glueballs (gluon-gluon bound states).The models are characterised by two scales: the confinement scale Λ_{DC} and the dark quark mass $M_{\mathcal{Q}}$. A large separation of scales can arise naturally between $M_{\mathcal{Q}}$ and Λ_{DC}, due to the renormalization group flow, if the dark colour gauge coupling at the scale $M_{\mathcal{Q}}$ is of order one or smaller.

The two models differ in the assignment of Standard Model quantum numbers for the dark quarks. They represent two benchmark scenarios: one without Yukawa couplings with the Higgs field in which three of the dark quarks transform as a triplet of $\mathrm{SU}(2)_{\mathrm{EW}}$, while the other two are singlets (we refer to this model as VNN using the notation of reference [34]); one with Yukawa interactions, in which four of the dark quarks transform as two doublets with opposite hypercharges and the last one transforms as a singlet (LLN model). For the two models, we have analysed how robust is the accidental stability of the neutral gluequarks: while in the VNN model the accidental symmetry protecting the gluequark from decaying is broken only at the level of dimension 6 operators, in the LLN model there exists a dimension 5 operator that induces the breaking of the accidental symmetry. In this case, in order to have a cosmologically stable dark matter candidate, it is necessary to have a small Yukawa coupling $y \lesssim 10^{-5}$. This dimension 5 operator is peculiar of theories with adjoint fermions and its existence depends crucially on the Standard Model representations of the dark quarks. This observation represents a crucial difference with respect to models in which the dark quarks transform in the fundamental representation [1,35]: in that case baryon number is violated at the level of dimension 6 operators, while dimension 5 operators induce only the decay of mesons, that would be stable thanks to species number and G-parity.

Subsequently, the phenomenology of the two models has been analysed. Depending on the two scales $\Lambda_{\rm DC}$ and M_Q the dynamics realises different regimes and different physical mechanisms become relevant; in particular we must treat separately the two cases in which the glueballs are stable or decay on cosmological scales. We have computed the glueball decay rate, identifying the relevant regimes. In the parameter space region with stable glueballs, we have computed the glueball relic density, including the effects of number changing interactions in the glueball sector that imply a non trivial cosmological evolution. Taking into account the constraints deriving from cosmological observations, we have concluded that a great portion of the parameter space is excluded by cosmological observations (in particular the whole region with stable glueballs and the region with glueballs decaying with a lifetime greater than 1 s).

One of the main results of the thesis is the calculation of the mass of the gluequark dark matter candidate as a function of the confinement scale $\Lambda_{\rm DC}$, as displayed in figure 5.1. We have computed the gluequark relic density, including the Sommerfeld enhancement correction, deriving the value M_Q of the dark quarks mass necessary to reproduce the correct thermal relic abundance. The calculation of the relic density that we have presented is limited to the case in which the freeze-out occurs before dark confinement. The computations give a dark matter candidate in the region of a few TeV, which seems very interesting from the phenomenological point of view. This result distinguishes our model from other models of composite dark matter, in which usually the mass of the dark matter candidate is heavier: $M_{\rm DM} \sim (10 \div 100)$ TeV.

Our study demonstrates that there are models based on a non-Abelian gauge group with a dynamics different from models realising vectorlike confinement and with peculiar properties, which can serve as viable dark sectors with accidentally stable dark matter candidates.

The dark matter candidate of the models with an infrared fixed point that we have studied is qualitatively different from dark baryons or dark mesons, being a composite state made of an heavy dark quark and a dark gluon. We plan to analyse the collider phenomenology of these models in further studies, assessing the reach of ongoing and future experiments for the detection of gluequarks and glueballs, and the bounds deriving from electroweak precision observables and electric dipole moments. A key point of this analysis will be the identification of characteristic signatures for this composite dark matter candidate, that in case of detection can discriminate its nature from that of other candidates.

Moreover, some refinement to the calculation of the relic density would be needed. Our calculation does not include the effect of bound state formation prior to annihilation [127] nor the annihilation of gluequarks after dark confinement, which could be relevant if the annihilation cross section becomes enhanced after confinement. The case in which dark confinement and freeze-out occur at the same epoch needs a separate analysis; while, in the case in which freeze-out occurs after confinement, the freeze-out mechanism should be modified, due to the unstable nature of the glueballs in which the gluequarks annihilate, realising a variation of the *one-way phase* described in [100].

From the view point of model building, the models we have considered have dark quarks transforming in Standard Model representations which are fragments of SU(5) representations, following [34] and assuming an SU(5) unification scheme at high energies. Since the dimension of the non-renormalizable operators that violate dark parity (giving strong constraints on the Yukawa couplings) depends crucially on the Standard Model quantum numbers, one could try to use different representations, embedding the theory in a different great unification scheme. This could allow, for instance, to have a model with Yukawa couplings but with no dimension 5 dark parity violating operators.

Finally, our analysis has been limited to the case of perturbative fixed points. It would be interesting to analyse a model for a dark sector based on a strongly interacting infrared fixed point and see which properties generalise to this case and which are the new features.

As for models with chiral representations, following this work, one point that we would like to clarify is what are the observational constraints on the light states required by 't Hooft anomaly matching. Regarding explicit models for a chiral dark sector, due to the difficulties in simulating chiral gauge theories on the lattice, there is no firm result on what is their infrared dynamics. One possibility is to make some dynamical assumptions and study the phenomenology that results, trying to build explicit models in which all the asymptotic states acquire a non-zero mass. In particular, we would like to build a chiral model of a dark sector with fermions charged under both $\mathcal{G}_{\mathcal{DC}}$ and $\mathcal{G}_{\mathcal{SM}}$ and to study its phenomenology, understanding if there can be a viable dark matter candidate and what are its peculiar properties.

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